SOM-LS: Selective Orthogonal Matrix Least-Squares Method for Macromodeling Multiport Networks Characterized by Sampled Data

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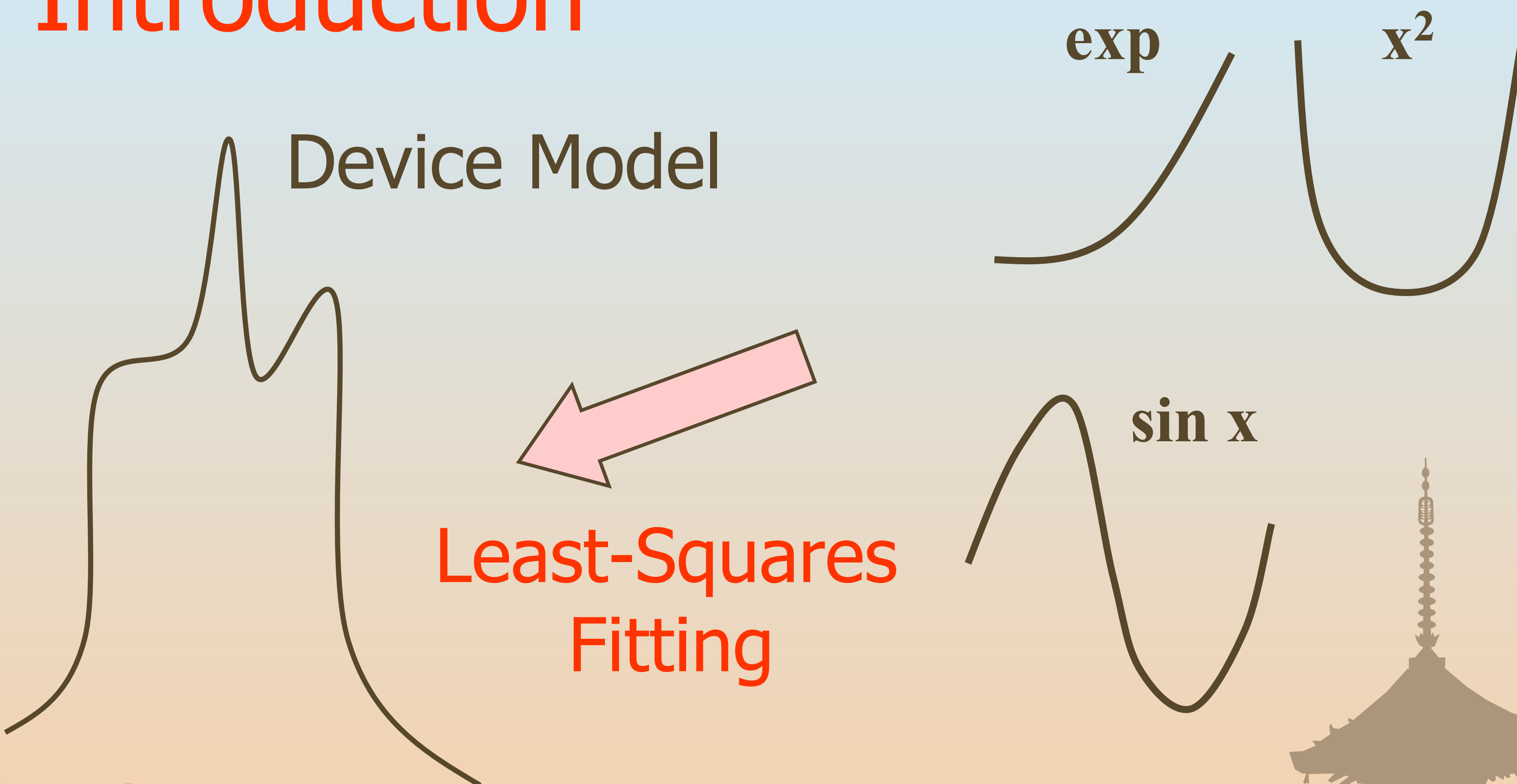
Introduction



- Which function is dominant ?

- The least-squares method for system identification selecting the dominant basis functions is presented.
- This method is based on a learning algorithm of neural network.
- Macromodeling of networks characterized by sampled data via electromagnetic analysis.
- The macromodels are described in the format of Verilog-A.

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Selective Orthogonal Matrix Least-Squares Method

Device Model:

$$\mathbf{G}(x) = \sum_{i=1}^{N} \mathbf{K}_i f_i(x)$$

$\mathbf{K}_i$: constant matrix

$f_i(x)$: basis function

x : design variable

- The device model is made by least-squares ▯fitting of sampled data

Over-determined Equation:

$$\mathbf{PK} = \mathbf{F}$$

$$\mathbf{P} = \begin{bmatrix} f_1(x_1) & f_2(x_1) & \cdots & f_M(x_1) \\ f_1(x_2) & f_2(x_2) & \cdots & f_M(x_2) \\ \vdots & \vdots & \cdots & \vdots \\ f_1(x_N) & f_2(x_N) & \cdots & f_M(x_N) \end{bmatrix}$$

$$\mathbf{F} = \left[\mathbf{G}(x_1), \mathbf{G}(x_2), \ldots, \mathbf{G}(x_N)\right]^T$$

- The over-determined matrix equation is solved by orthogonal least-squares method.

- The coefficient matrix is rewritten by

$$\mathbf{P} = [\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3, \mathbf{p}_4, \ldots, \mathbf{p}_k, \ldots, \mathbf{p}_N]$$

- The number of basis functions is equal to the number of orthogonal vectors
- The key issue is how to select the column vectors and orthogonalize them.

$$\mathbf{P} = [\mathbf{w}_1, \mathbf{w}_2, \mathbf{w}_3, \mathbf{w}_4, \ldots \mathbf{w}_{k-1}, \mathbf{p}_k, \ldots, \mathbf{p}_N]$$

$$\mathbf{P} = [\mathbf{p}_1, \ldots, \mathbf{p}_{N-k+1}, \mathbf{w}_1, \mathbf{w}_2, \mathbf{w}_3, \mathbf{w}_4, \ldots \mathbf{w}_{k-1}]$$

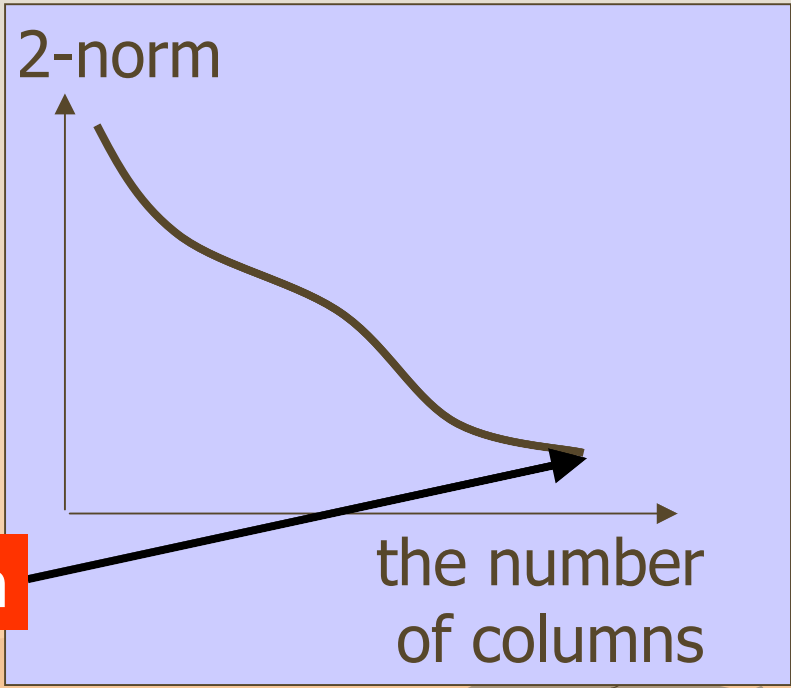
- After *k*th step in the orthogonalization, the residual matrix is defined as

$$\mathbf{Z} = \mathbf{F} - \mathbf{W}_k \mathbf{G}_k$$

$\mathbf{W}_k$: orthogonal matrix

$\mathbf{G}_k$: intermediate solution

- The 2-norm of the residual matrix is proven to be monotonously decreasing function.

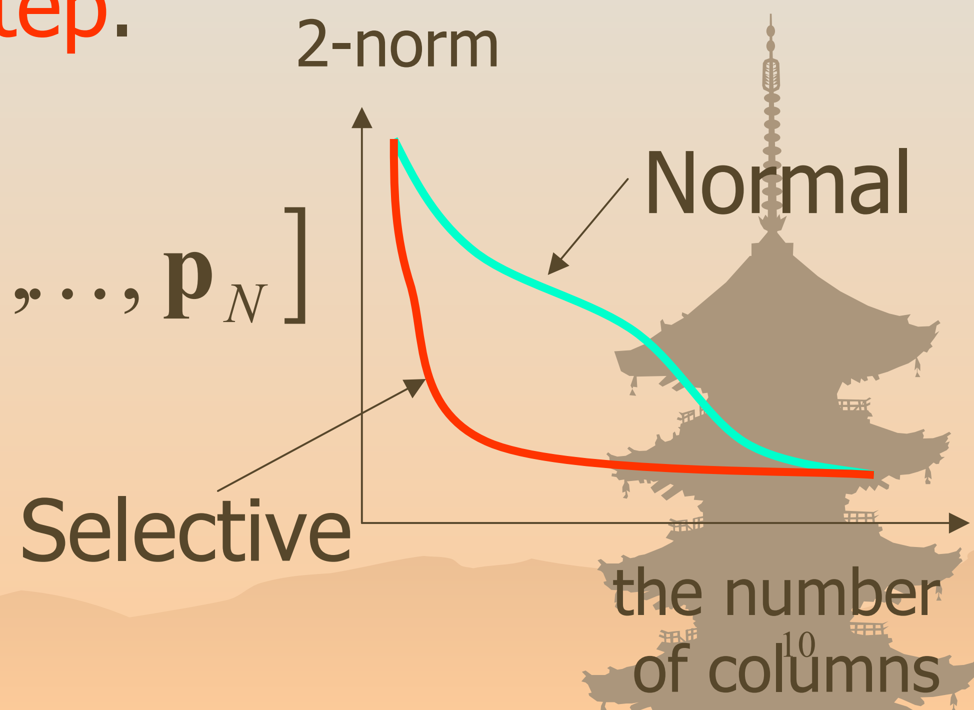


Least Squares Solution

Selective Orthogonalization

- Evaluating the 2-norm of the residual matrix, the columns of the matrix **P** are orthogonalized so that the 2-norm largely decreases at each step.

$$\mathbf{P} = \left[\mathbf{w}_1, \mathbf{w}_2, \mathbf{p}_3, \ldots, \mathbf{p}_k, \ldots, \mathbf{p}_N\right]$$



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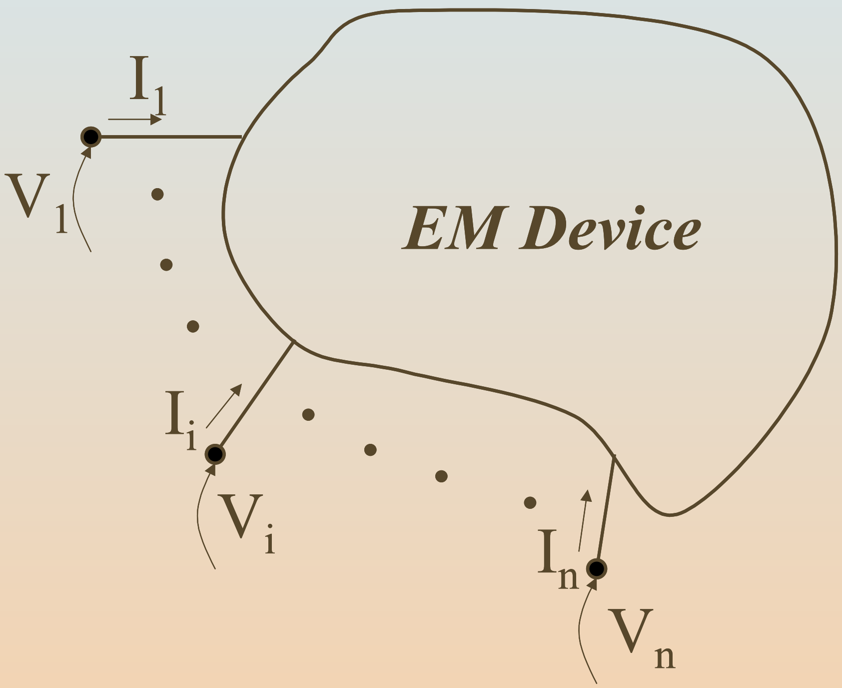
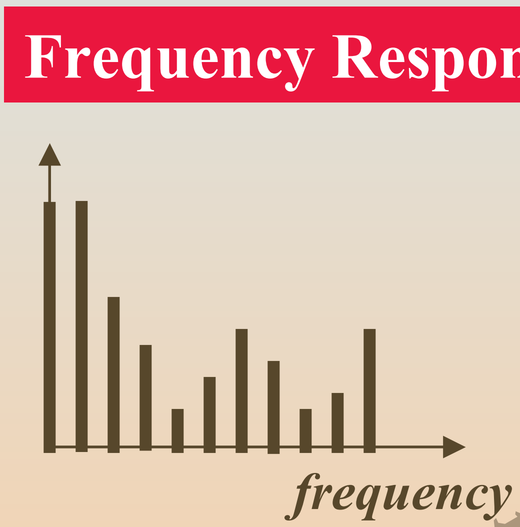
Macromodeling of Networks Characterized by Sampled Data





- The SOM-LS method is used for approximating the sampled data with the rational matrix.

1st Level Approximation

Sampled Data

↓

Rational Function

↓

Stable Poles

$$Y_{ij}(j\omega_i) = \frac{b_0 + b_1(j\omega_i) + \cdots + b_m(j\omega_i)^m}{1 + a_1(j\omega_i) + \cdots + a_n(j\omega_i)^n}$$

$$(i = 1, \cdots, N)$$

Least-Squares Fitting
(scalar approximation)

Root Finding

2^{nd} Level Approximation

Sampled Data

↓

Rational Matrix

$$\sum_{l=1}^{Q} \frac{\mathbf{K}_l}{j\omega_i - p_l} = \mathbf{Y}(j\omega_i)$$

$$(i = 0, 1, \ldots, N)$$

Least-Squares Method
(Matrix Approximation)

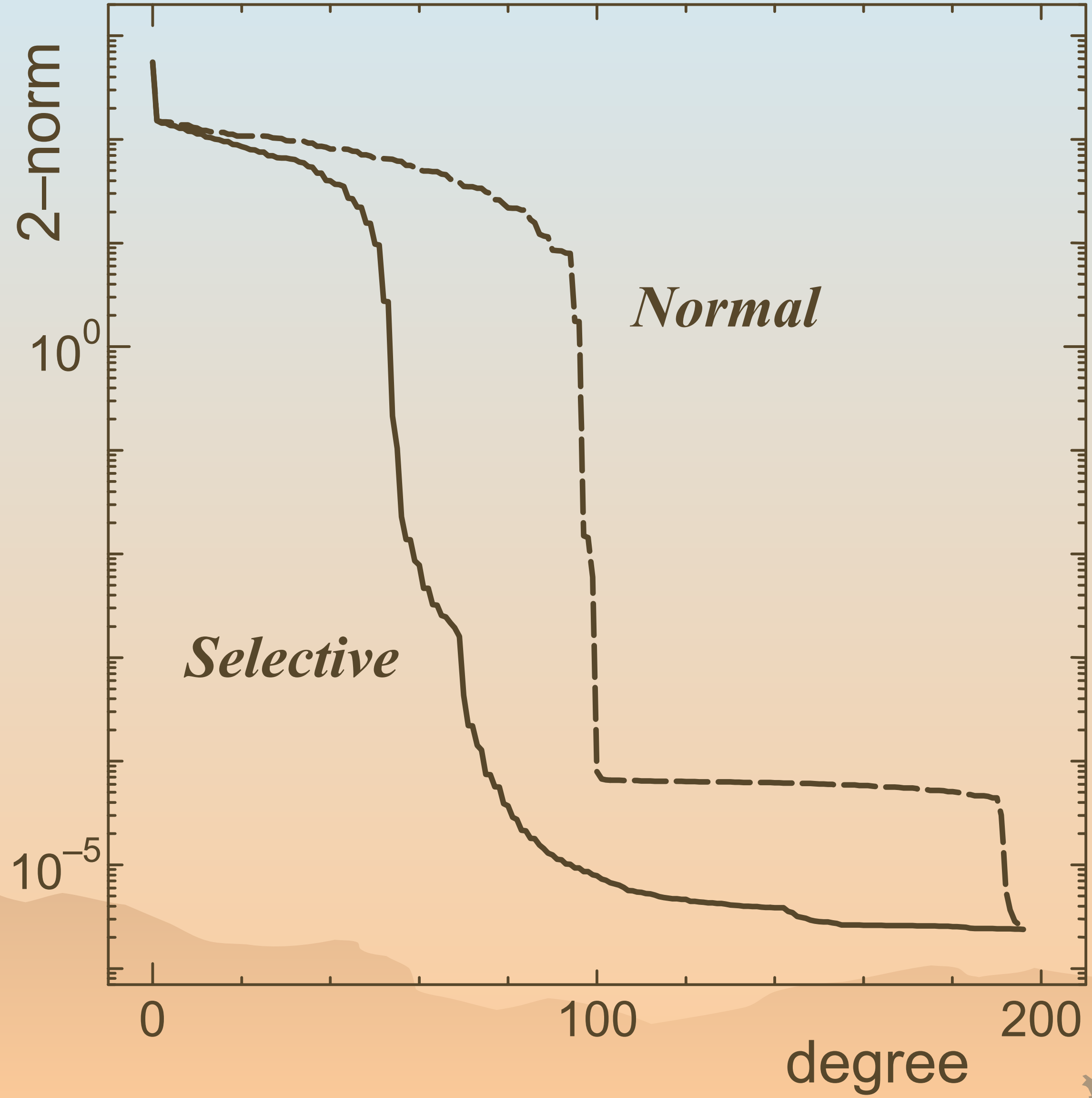
- Using the SOM-LS method, the dominant poles are extracted and the compact model is obtained.

Orthogonal Least-Squares Method

$$\mathbf{PK} = \mathbf{F}$$

$$\mathbf{P} = \begin{bmatrix} 1 & \dfrac{1}{j\omega_1 - p_1} & \cdots & \dfrac{1}{j\omega_1 - p_Q} \\ 1 & \dfrac{1}{j\omega_2 - p_1} & \cdots & \dfrac{1}{j\omega_2 - p_Q} \\ \vdots & \vdots & \cdots & \vdots \\ 1 & \dfrac{1}{j\omega_N - p_1} & \cdots & \dfrac{1}{j\omega_N - p_Q} \end{bmatrix}$$

2-norm of Residual Matrix



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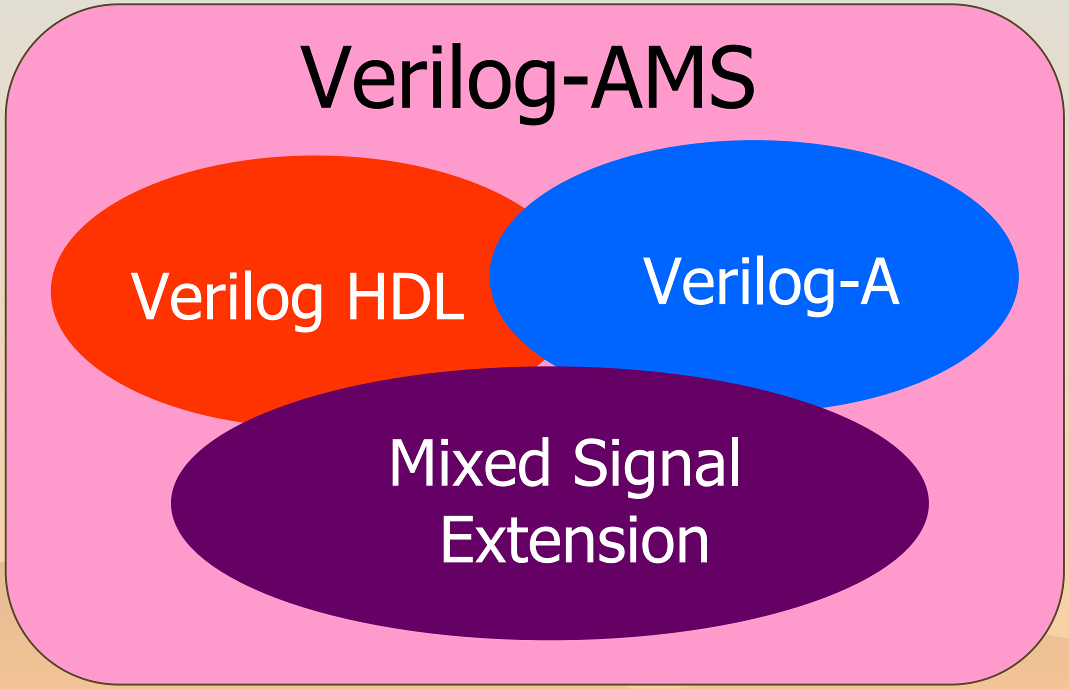
Examples

- The marcromodels are described by Verilog-A.



Laplace Transform Description by Verilog-A

$$\left.\begin{aligned} V_{out}(s) &= H(s)\cdot V_{in}(s) \\ H(s) &= \frac{b_0 + b_1\cdot s}{a_0 + a_1\cdot s + a_2\cdot s^2} \end{aligned}\right\}$$

```
module transfer_func(in, out);

inout in, out;
electrical in, out;

analog begin

    V(out) <+ laplace_nd(V(in), [b0,b1], [a0, a1, a2]);

end

endmodule
```

Verilog-A Model Generation

pole-1(real) pole-1(imag) residue-1(real) residue-1(imag)

pole-2(real) pole-2(imag) residue-2(real) residue-2(imag)

:

pole-

module model_name(in, out);

inout in, out;
electrical in, out;

analog begin

V(out) <+ laplace_nd(V(in), [b_0,b_1], [a_0, a_1, a_2]);

V(out) <+ laplace_nd(V(in), [b_0,b_1], [a_0, a_1, a_2]);

:

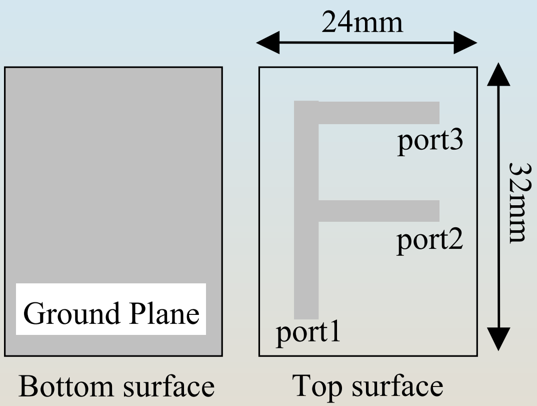
V(out) <+ laplace_nd(V(in), [b_0,b_1], [a_0, a_1, a_2]);

end

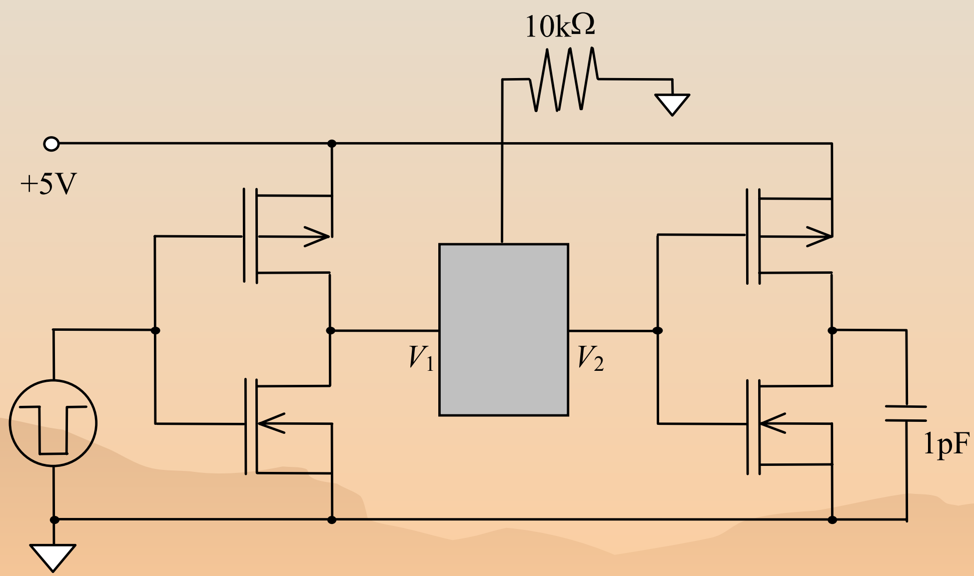
endmodule

- We computed the responses of simple PCB models using *Cadence Spectre*.
- The results using the proposed macromodels were compared with the FDTD method on *Spectre*.
- The computational speed with the proposed macromodels is two magnitudes faster than the FDTD method on *Spectre*.

Example PCB Model

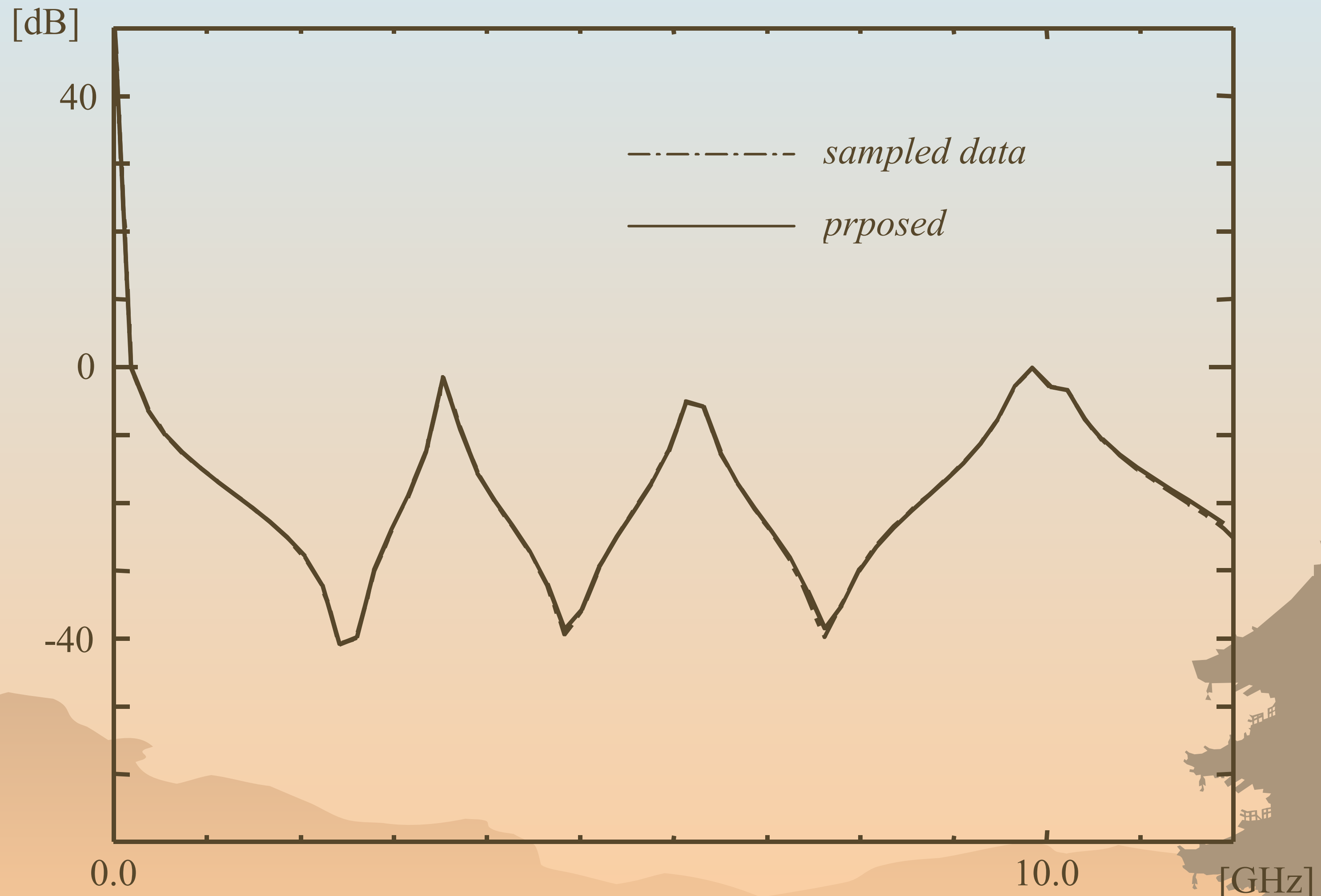


(a)

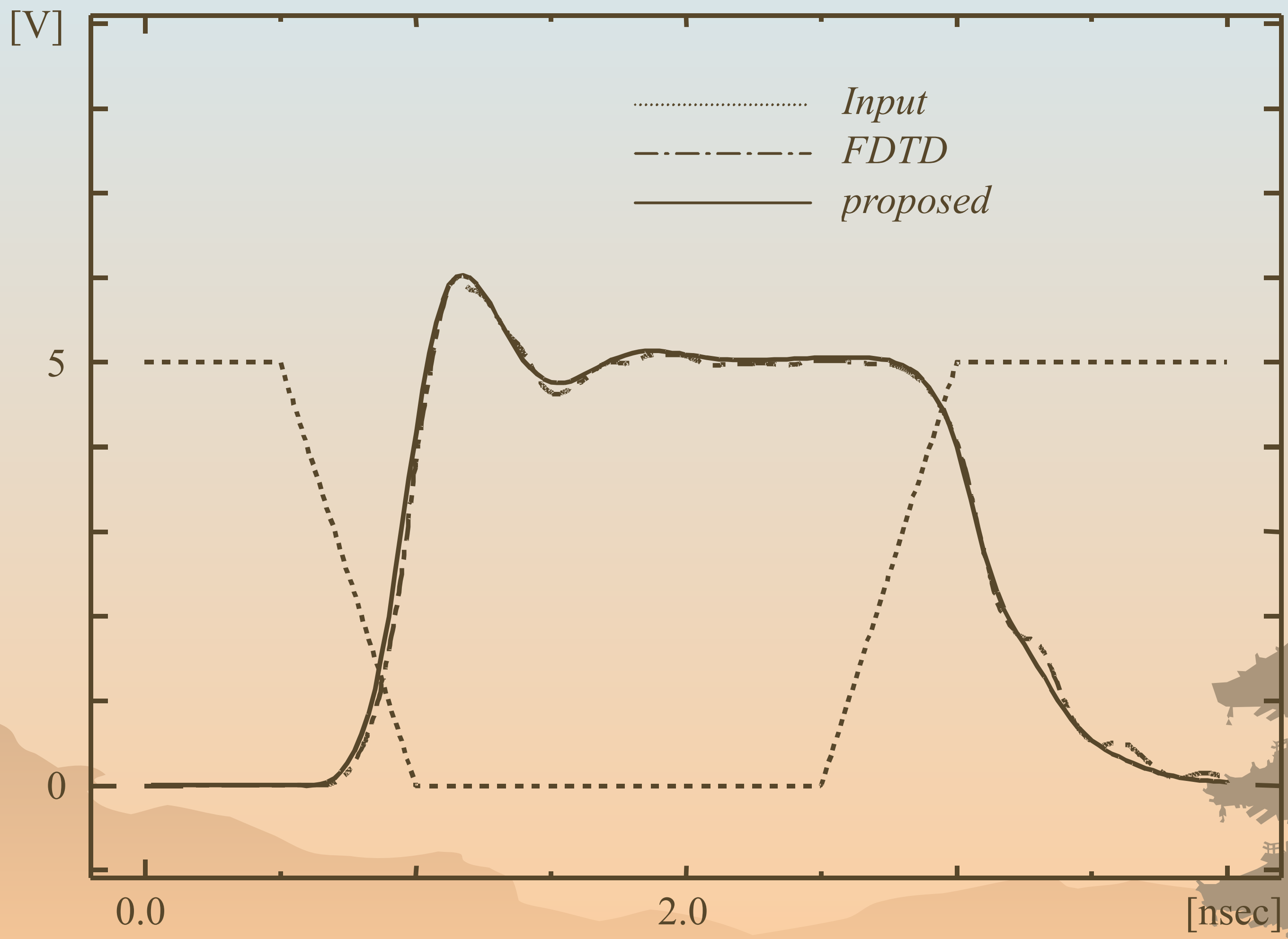


(b)

Frequency-Domain Response



Time-Domain Response



CPU Time Comparison

	FDTD	Proposed Model
Example1	550.94 (sec)	4.27 (sec)
Example2	596.19 (sec)	5.96 (sec)
Example3	416.96 (sec)	4.09 (sec)

Summary

- The selective orthogonal matrix least-squares method is presented.
- This method is applied to macromodeling of networks characterized by sampled data.
- The proposed models are described in the format of Verilog-A.
- Future work: Passivity consideration of the macromodel.