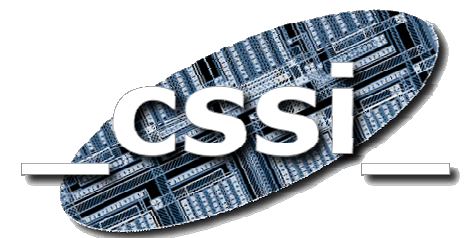




A Methodology for Analog Circuit Macromodeling

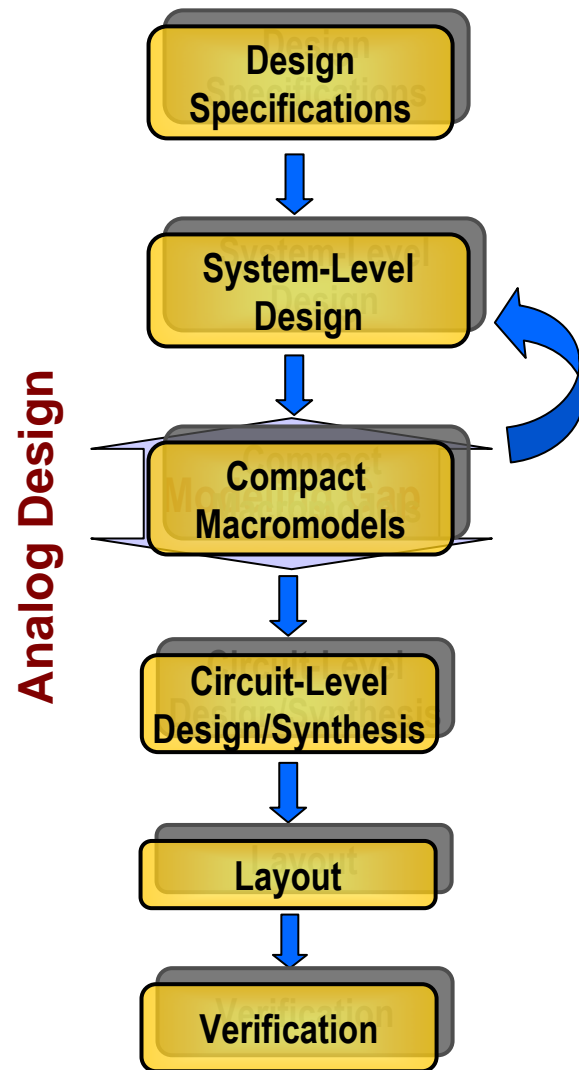
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Motivation

- ❑ Compact sub-block macromodels are the key to whole-system verification
- ❑ Back-annotation of such models facilitates system-level verification
- ❑ These “reduced-order” macromodels capture the nonlinear effects
 - IIP3, THD, gain compression,...
 - Dynamic range, spectral regrowth, etc...



Agenda

- **Introduction**
 - ✓ Motivation
 - **Previous Work**

- **Nonlinear macromodeling approach**
 - **Background**
 - **Extraction of Volterra Parameters**
 - **Overall Macromodeling Flow**

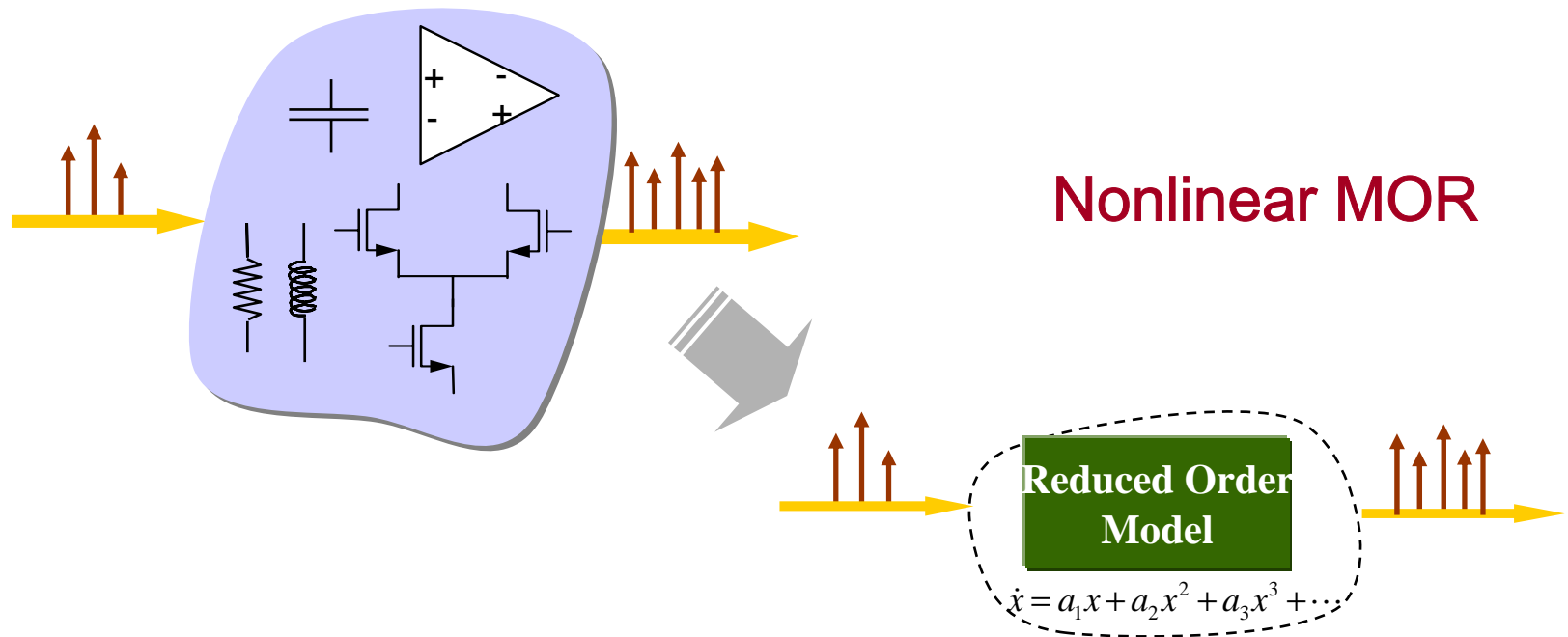
- **Experimental results**
- **Conclusions**

Previous Work

- ❑ **Reduced order modeling of time-varying systems**
 - [Roychowdhury TCAS 1999]
 - [Phillips CICC 2000]
- ❑ **PWL/PWP and model order reduction**
 - [Rewienski, White ICCAD 2001]
 - [Dong, Roychowdhury DAC 2003]
- ❑ **NORM : compact model order reduction of weakly nonlinear systems**
 - [Li, Pileggi DAC 2003]
- ❑ **Hybrid approach to nonlinear macromodel generation**
 - [Li, Xu, Li, Pileggi ICCAD 2003]
- ❑ **Multivariate formulation**
 - [Li, Pileggi BMAS 2003]

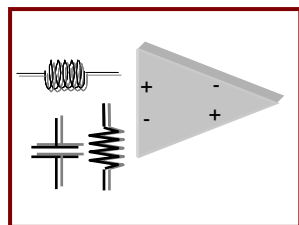
Macromodeling Problem

- Can we build efficient analog macromodels to capture: *linear conversion + time variance + distortion* ?



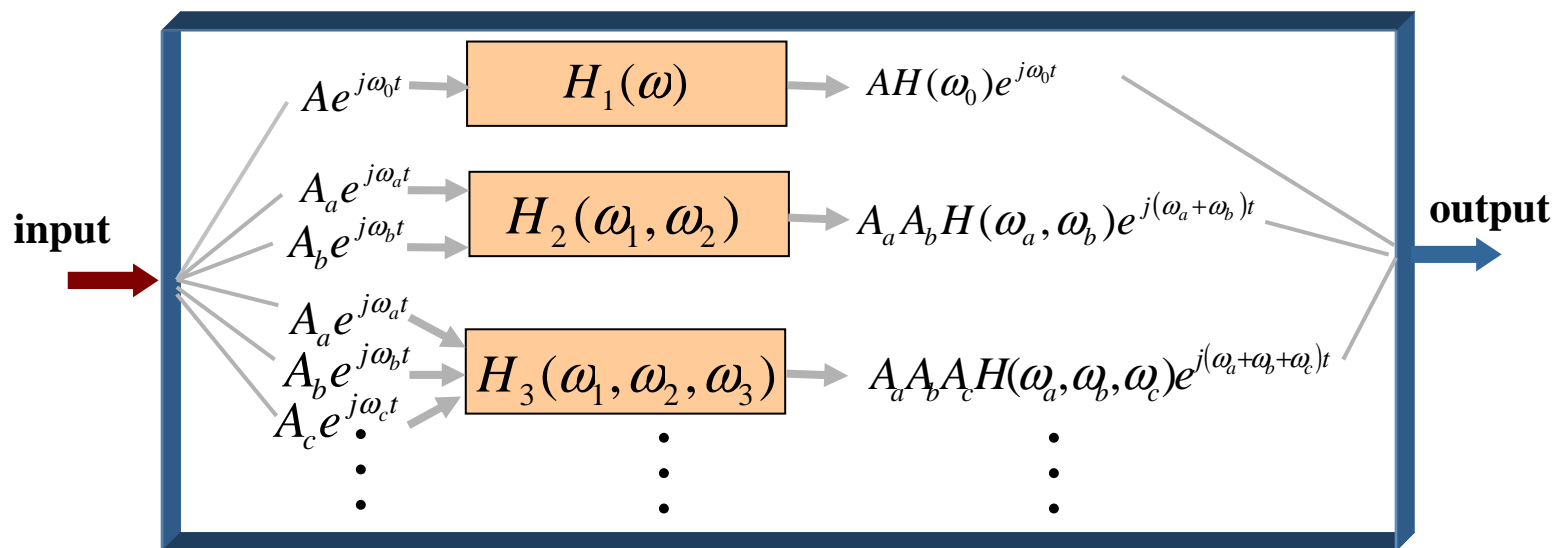
Modeling of Nonlinear Analog Circuits

- Volterra Series to describe weakly nonlinear systems



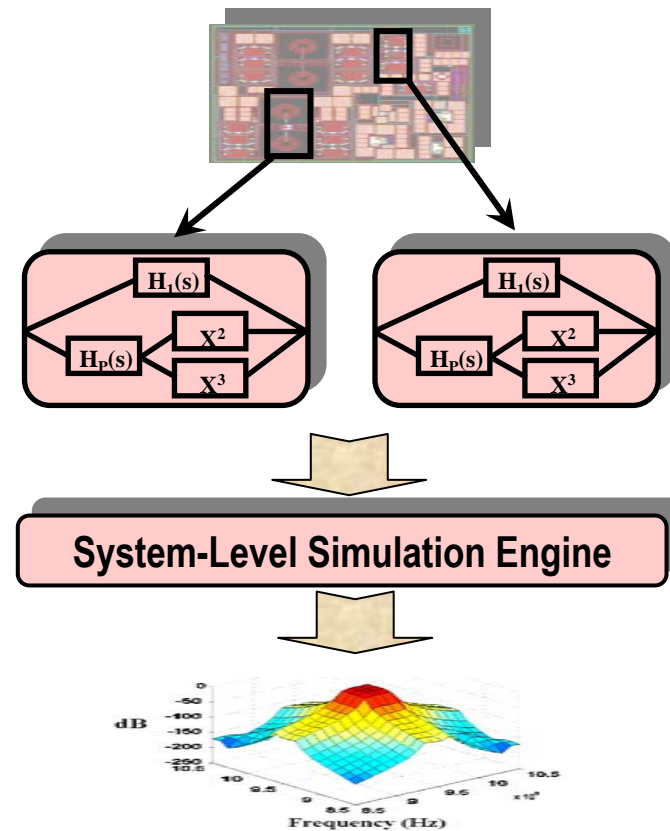
$$x(t) = \sum_{n=1}^{\infty} x_n(t)$$

$$H_{x_n}(f) \doteq \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \dots \int_{-\infty}^{\infty} h_{x_n}(\tau_1, \dots, \tau_n) u(t - \tau_1) \dots u(t - \tau_n) e^{-j2\pi(f_1\tau_1 + \dots + f_n\tau_n)} d\tau_1 \dots d\tau_n$$



Modeling of Weakly Nonlinear Circuits

- ❑ Volterra-based descriptions are based on multi-dimensional transfer functions
- ❑ Full-models are extremely complex to be used efficiently
- ❑ Reduced-order modeling techniques are required



Nonlinear Transfer Functions

□ Nonlinear dynamic system

$$f \frac{d}{dt}(C_1 x) + C_2 \left(\frac{d}{dt}(x \otimes x) \right) + C_3 (x \otimes x \otimes x) + \dots + G_1 x + G_2 (x \otimes x) + G_3 (x \otimes x \otimes x) + \dots = bu$$

$$G_i = \left. \frac{1}{i!} \frac{\partial^i}{\partial x^i} (f) \right|_{x=x_0} \quad C_i = \left. \frac{1}{i!} \frac{\partial^i}{\partial x^i} (q) \right|_{x=x_0}$$

$$x \otimes x = [x_1^2 \quad x_1 x_2 \quad \dots \quad x_1 x_n \quad \dots \quad x_n x_1 \quad \dots \quad x_n^2]^T$$

$$x \otimes x \otimes x = [x_1^3 \quad x_1^2 x_2 \quad \dots \quad x_1 x_n^2 \quad \dots \quad x_n x_1^2 \quad \dots \quad x_n^3]^T$$

□ 1st order

$$H_1(s) = (SC_1 + G_1)^{-1} b$$

□ 2nd order

$$(s_1 + s_2)C_1 \cdot H_2(s_1, s_2) + G_1 \cdot H_2(s_1, s_2) = -((s_1 + s_2)C_2 + G_2) \cdot \overline{H_1(s_1) \otimes H_1(s_2)}$$

$$\overline{H_1(s_1) \otimes H_1(s_2)} = \frac{1}{2} [H_1(s_1) \otimes H_1(s_2) + H_1(s_2) \otimes H_1(s_1)]$$

Moments of Nonlinear Transfer Functions

- Moments of linear transfer functions

$$H(s) = L^T (sC + G)^{-1} B$$

$$H(s) = M_0 + M_1 s + M_2 s^2 + \dots$$

- Now for nonlinear transfer functions

$$H_2(s_1, s_2) = M_{2,0} + \underbrace{M_{2,1,1}s_1 + M_{2,1,0}s_2}_{1^{\text{st}} \text{ order}} + \underbrace{M_{2,2,2}s_1^2 + M_{2,2,1}s_1s_2 + M_{2,2,0}s_2^2}_{2^{\text{nd}} \text{ order}} + \dots$$

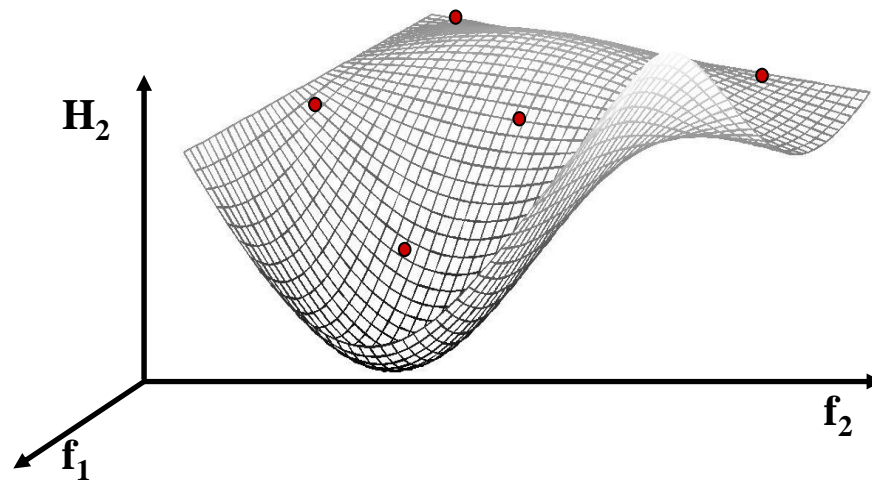
$$H_3(s_1, s_2, s_3) = M_{3,0} + \underbrace{M_{3,1,1,0}s_1 + M_{3,1,0,1}s_2 + M_{3,1,0,0}s_3}_{1^{\text{st}} \text{ order}} + \underbrace{M_{3,2,2,0}s_1^2 + M_{3,2,1,1}s_1s_2 + \dots + M_{3,2,0,0}s_3^2}_{2^{\text{nd}} \text{ order}} + \dots$$

- Moment matching these nonlinear transfer fcts via projection for MOR

Reduced Order Modeling

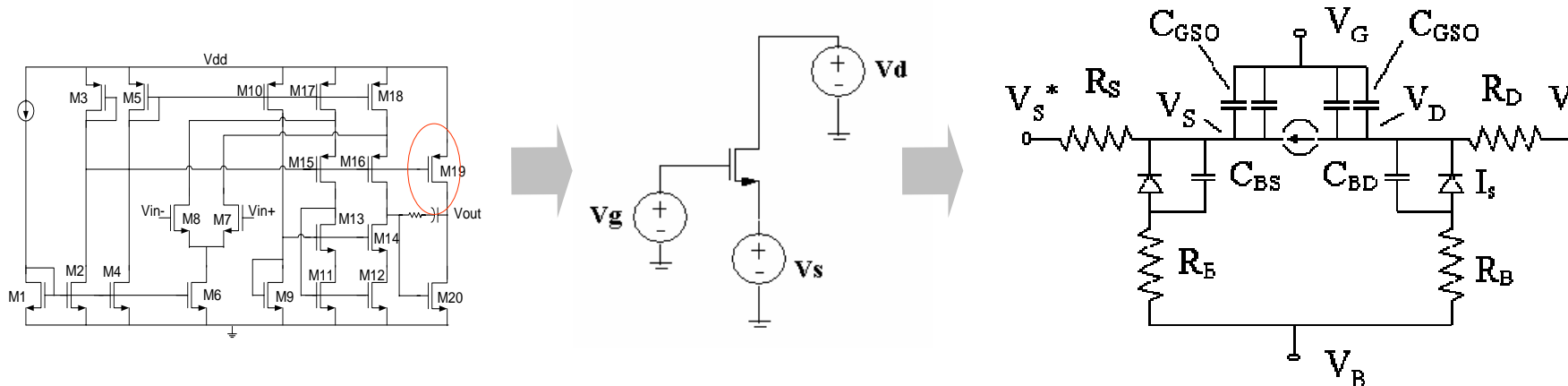
- Using a projection-based reduction – multipoint NORM

$$\tilde{G}_1 = V^T \bar{G}_1 V \quad \tilde{C}_1 = V^T \bar{C}_1 V \quad \tilde{b} = V^T \bar{b} \quad \tilde{G}_2 = V^T \bar{G}_2 (V \otimes V) \quad \tilde{C}_2 = V^T \bar{C}_2 (V \otimes V)$$



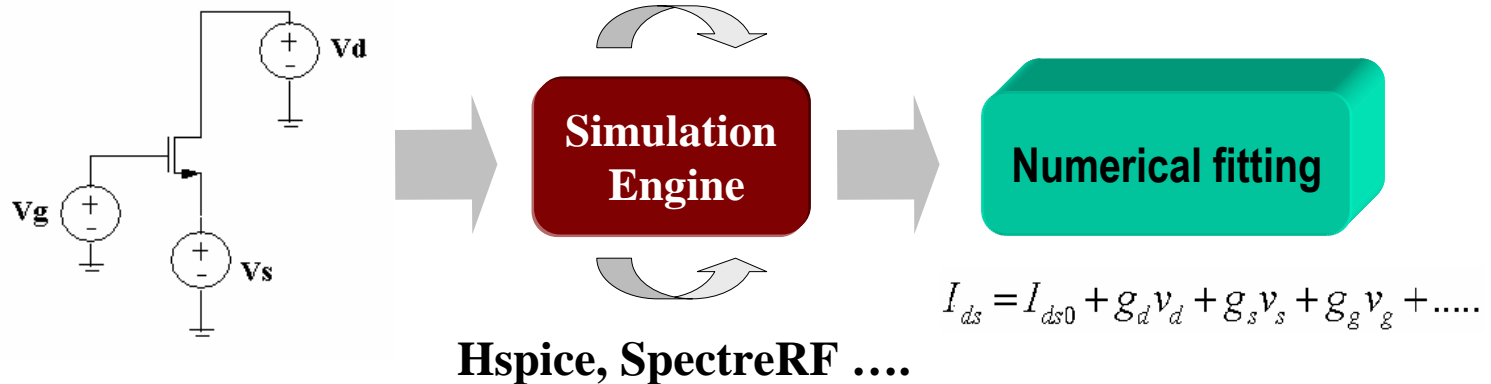
- Fully captures interactions between transfer functions of different orders

Extraction of Volterra Parameters



- ❑ Spice models like BSIM3 include physical effects + numerical parameters which increase model complexity
- ❑ Infeasible to determine the coefficients by computing higher derivatives of device model equations

Simulation Setup



- ❑ Commercial simulators like Hspice, SpectreRF can be used to characterize the model parameters for each transistor
- ❑ For each transistor, perturb the bias voltages to generate data-points for numerical fitting

Least-squares fitting

- Y matrix contains the voltage powers and cross terms
- p contains the corresponding coefficients
- R contains the residue ($I_{ds} - I_{ds0}$)

$$Y = \begin{bmatrix} v_{d1} & v_{s1} & v_{g1} & v_{d1}^2 & v_{d1}v_{g1} & \dots\dots\dots v_{s1}^3 \\ v_{d2} & v_{s2} & v_{g2} & v_{d2}^2 & v_{d2}v_{g2} & \dots\dots\dots v_{s2}^3 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ v_{dn} & v_{sn} & v_{gn} & v_{dn}^2 & v_{dn}v_{gn} & \dots\dots\dots v_{sn}^3 \end{bmatrix},$$

$$p = [g_d \quad g_s \quad g_g \quad g_{dg} \quad \dots\dots\dots g_{sss}]$$

$$R = [I_1 \quad I_2 \quad \dots\dots I_n]^T \quad I_n = I_{dsn} - I_{ds0}$$

Least-squares fitting

- Minimize the error for each data-point, in matrix form:

$$e = [\varepsilon_1 \ \varepsilon_2 \ \dots \ \varepsilon_n]^T$$

$$Yp - R = e$$

- The least-mean-square algorithm estimates the coefficients in p by minimizing the sum of squares errors:

$$F = e^T e = (Yp - R)^T (Yp - R)$$

$$p = (Y^T Y)^{-1} \cdot (Y^T R)$$

Improving the fit ...

□ Weighted-least squares approach

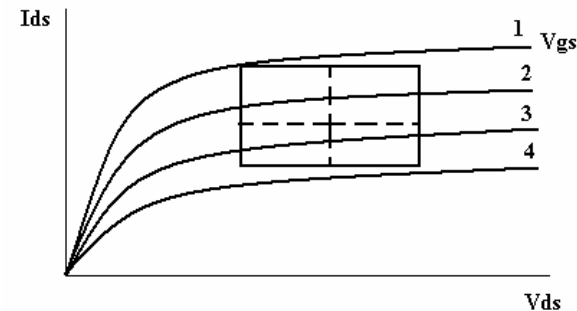
$$w_i(I_{dsi} - I_{ds0}) = g_d v_{di} w_i + g_s v_{si} w_i + \dots + g_{ds} v_{di} v_{si} w_i + \dots$$

$$F = (Yp - R)^T W (Yp - R)$$

$$p = (Y^T W Y)^{-1} \cdot (Y^T W R)$$

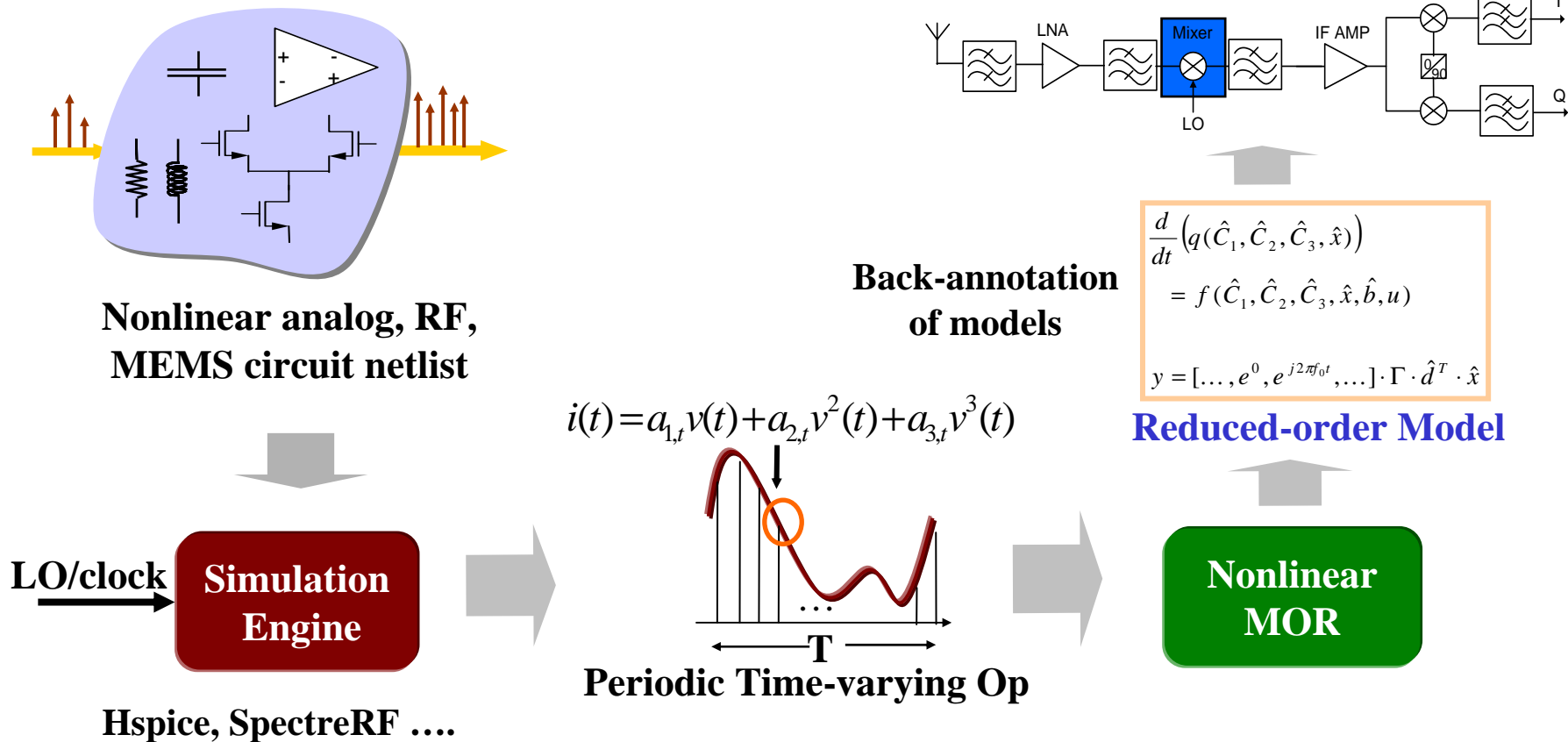
□ Fitting range

- **Large enough to encompass nonlinearities**
- **Should not cover effects outside signal swing range***



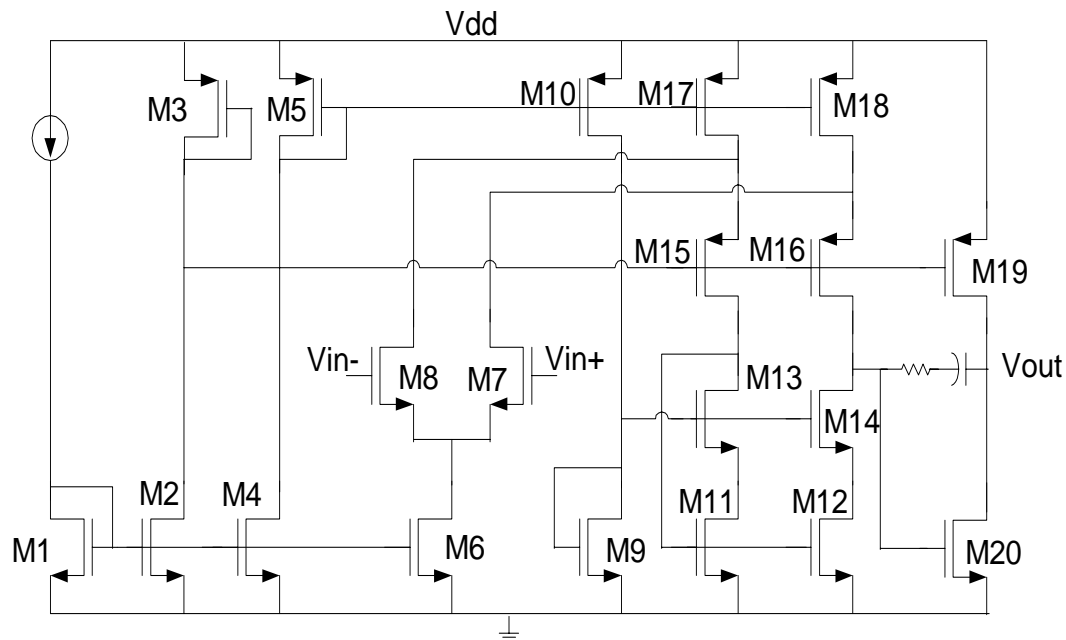
* ["Distortion in RF power amplifiers", Vuolevi and Rahkonen, Artech House, 2003]

Overall Macromodeling Flow



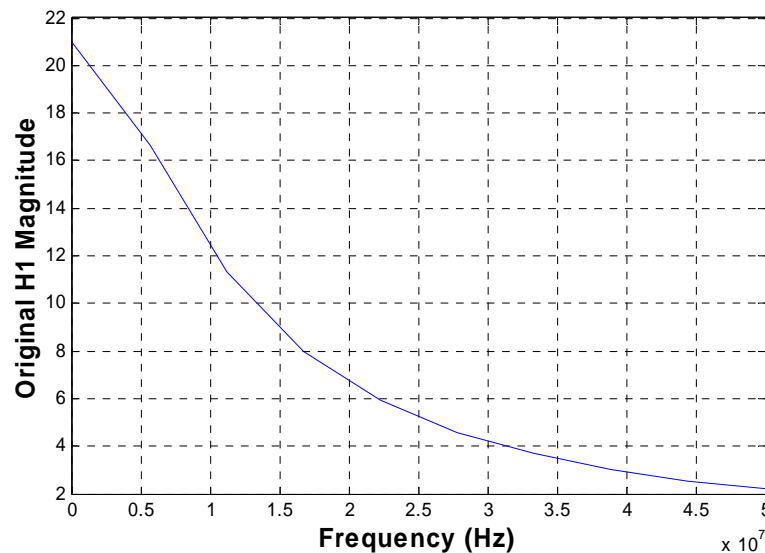
An Opamp

- ❑ Modeled as a time-invariant system, linearized at the DC bias point to fit second and third-order coefficients for each transistor
- ❑ Second-order nonlinearities are much higher than third-order nonlinearities for single-ended output



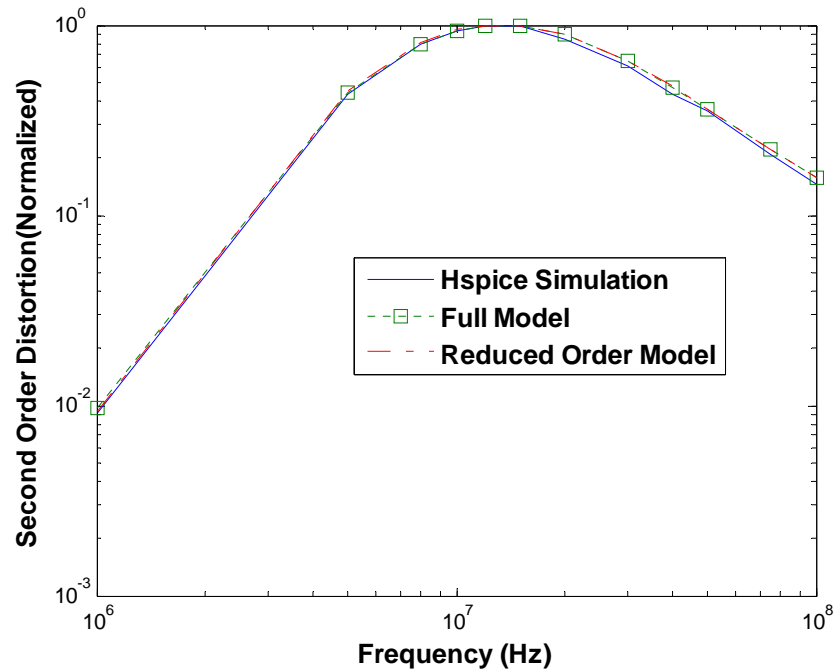
An Opamp

- ❑ Perform transient analysis in Hspice followed by fourier transform to compare with model results
- ❑ First-order (small-signal) results
 - Max error between full model generated using small-signal parameters and reduced-order model is 0.07%



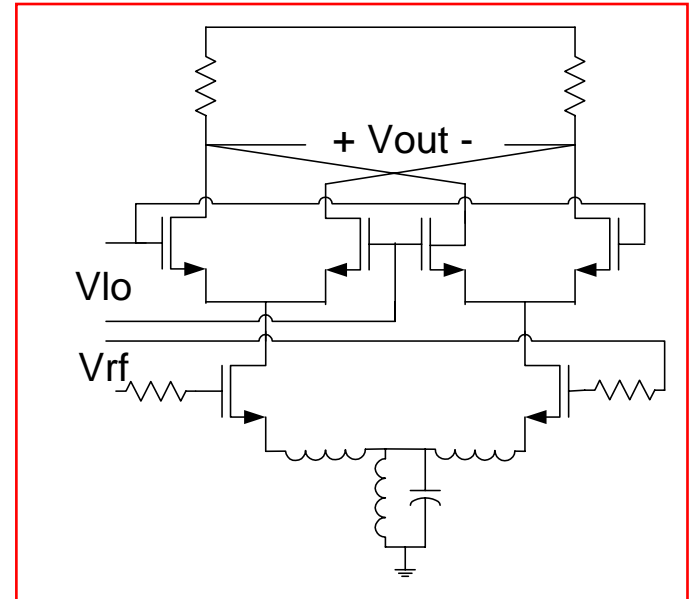
An Opamp

- **Second-order distortion as a function of frequency**
 - **Max error between transient simulation and full model : 5.1%**
 - **Max error between full model and reduced model : 2.9%**
 - **Max error between transient simulation and reduced model : 5.3%**

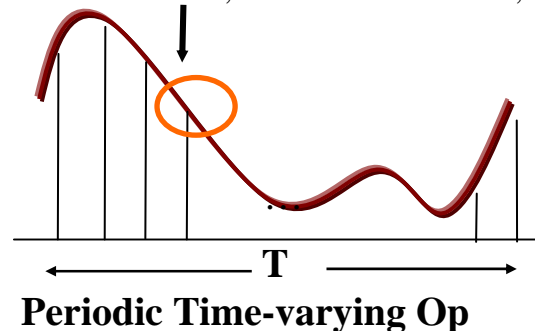


A Double-Balanced Mixer

- Characterized using time-varying Volterra series w.r.t. RF input based on 1350 time-sampled circuit variables
- Each nonlinearity is modeled as a third-order polynomial about the time varying operating point due to large-signal LO



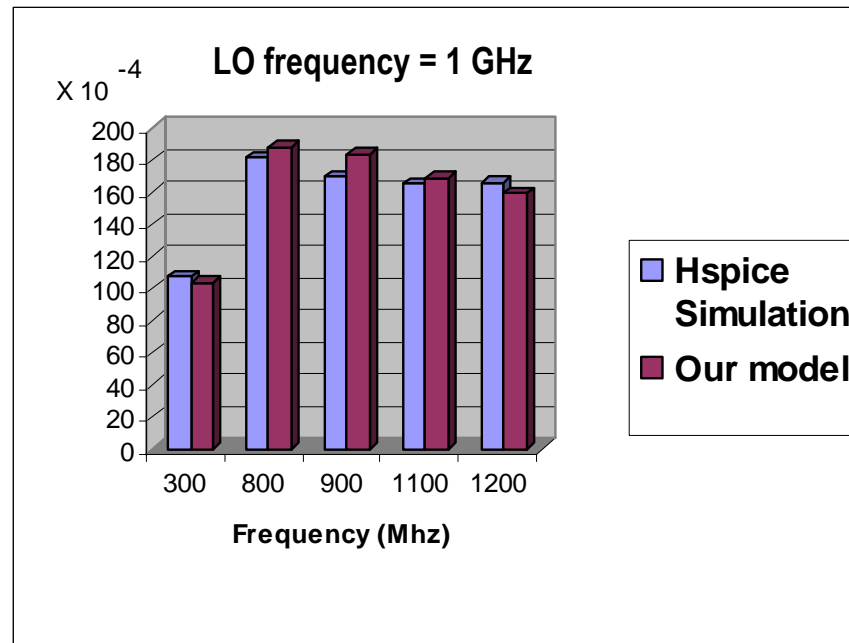
$$i(t) = a_{1,t}v(t) + a_{2,t}v^2(t) + a_{3,t}v^3(t)$$



A Double-Balanced Mixer

□ Third-order results

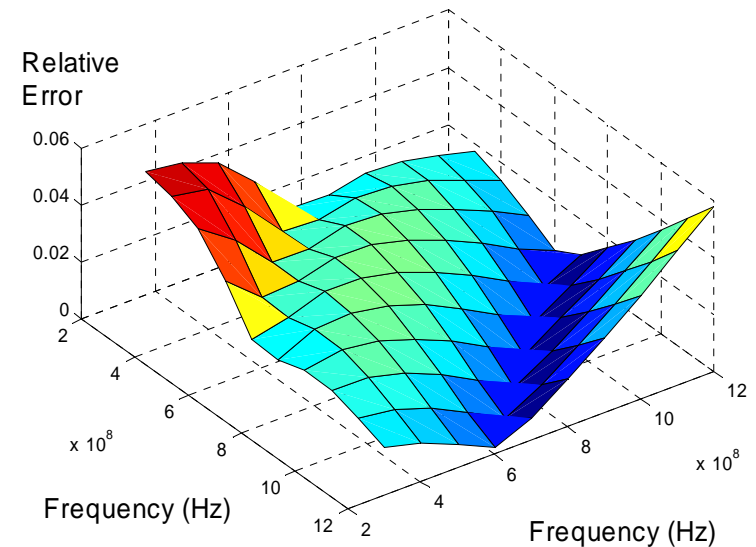
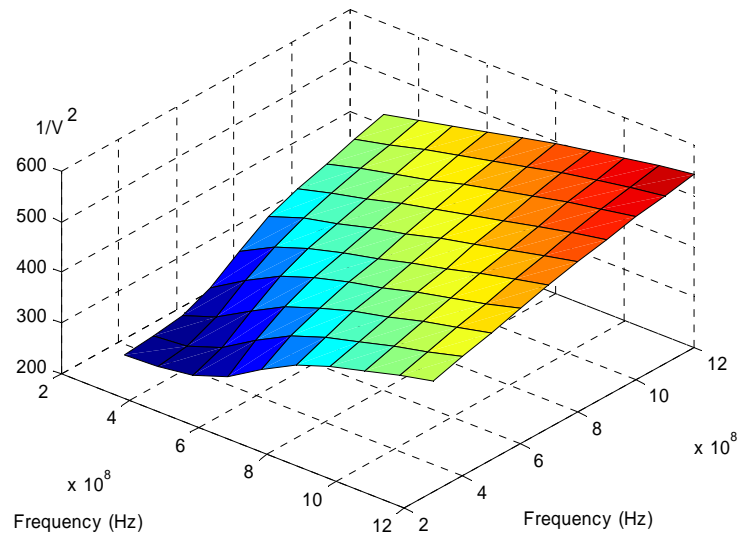
- Single tone RF input frequency varied from 300MHz to 1200MHz
- Third-order harmonic of the RF input down-converted w.r.t LO
- Max error between transient simulation and full model : 8%



A Double-Balanced Mixer

□ Reduced order model

- 14 circuit variables as compared to 1350 in the full model
- Third-order transfer function: $300\text{MHz} \leq f_1, f_2 \leq 1.2\text{GHz}$ and $f_{LO} = 1\text{GHz}$



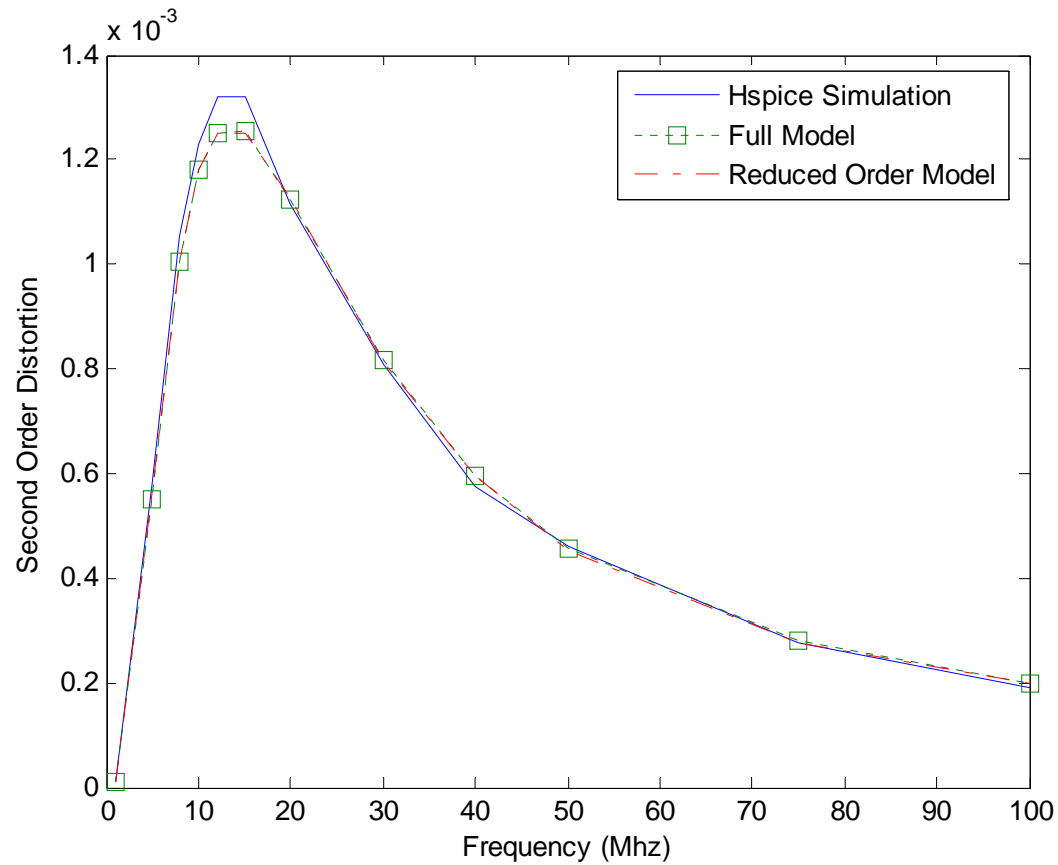
Conclusions

- ❑ **Reduced-order models for weakly nonlinear analog circuits can be generated from transistor-level netlists**
- ❑ **The accuracy is comparable to transistor-level simulation using commercial simulators**
- ❑ **Explore the adoption of these compact reduced-order models in behavioral languages like Verilog-A**



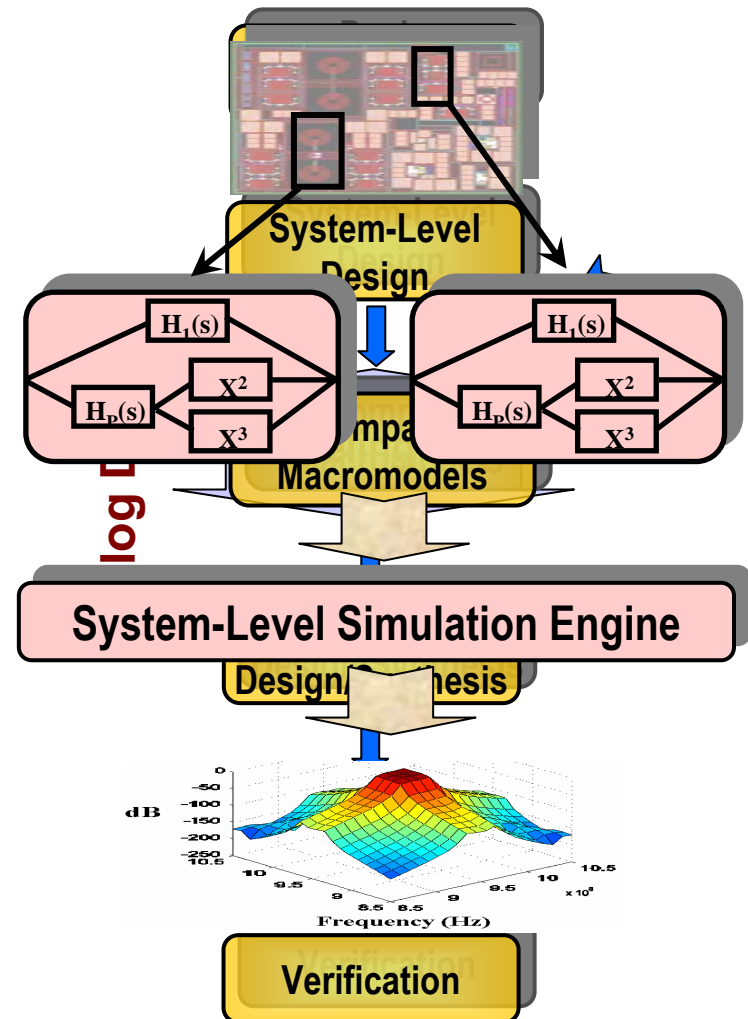
Backup Slides

Opamp Results



Compact Nonlinear Macromodels

- The purpose of developing compact macromodels is two-fold
 - A library of reduced-order models can facilitate system-level design exploration by “re-use”
 - Verification of the complete system based on these compact macromodels



Overall Macromodeling Flow

