

Variational Compact Modeling and Simulation for Linear Dynamic Systems



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Outline

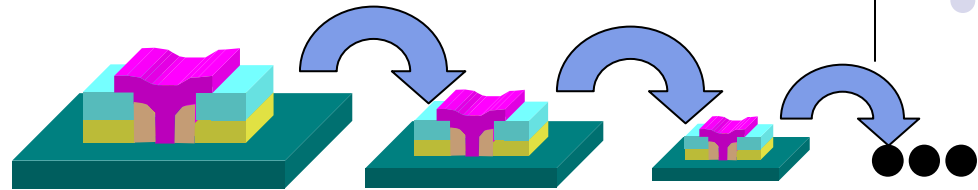
- Introduction
- Problem formulation
- Statistical based simulation method
 - Hermite polynomial chaos
 - Modified Krylov subspace model order reduction (MOR)
- Examples
 - One random variable
 - Multiple random variables
- SSMOR flowchart
- Experimental results
- Conclusion

Manufacturing Variations is REAL

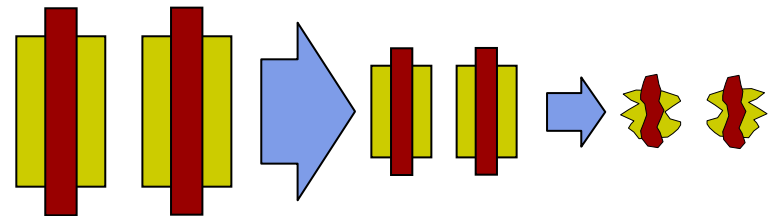


- CMOS scaling...

- Good for speed
- Good for density



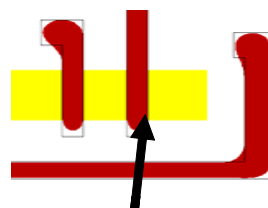
- Bad for variability
- Bad for manufacturability
- Bad for predictability



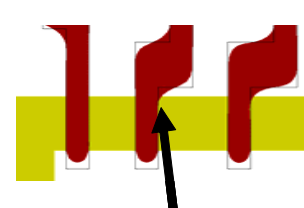
- What you design and verify pre-silicon is *not* the same as what you get post-silicon



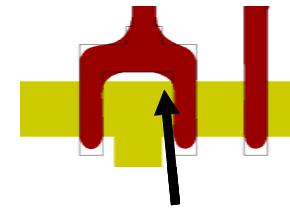
Sensitive to grow due to defocus



Sensitive to shrink due to defocus



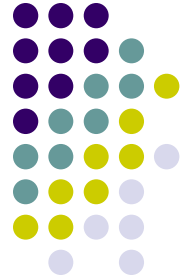
Sensitive to exposure variation



Sensitive to resist effects

[Source: Rob Rutenbar and Andrzej Strojwas, CMU]

Putting Variations in Context: Devices + Wires



- 3σ of variations normalized to nominal values [Nassif, CICC'2001]
- Manipulating things that are getting smaller
- Inevitable fluctuations are getting *relatively* larger
 - Variations on (spatially nearby) devices/wires are *correlated*

Year	$L_{\text{eff}} \pm 3\sigma$ (% nom)	$T_{\text{ox}} \pm 3\sigma$ (% nom)	$V_T \pm 3\sigma$ (% nom)	$W \pm 3\sigma$ (% nom)	$H \pm 3\sigma$ (% nom)	$\rho \pm 3\sigma$ (% nom)
1997	32%	8%	10%	25%	25%	22%
1999	33%	8%	10%	26%	30%	24%
2002	35%	10%	10%	28%	30%	27%
2005	40%	12%	11%	30%	34%	32%
2006	47%	16%	13%	33%	36%	33%



Review of existing approaches for statistical MOR methods



- Perturbation based MOR (L. Daniel, TCAD04, X. Li, ICCAD05)
 - – inter-die variation only
- Interval-valued modeling (J. Ma:ICCAD'04, ISPD'05)
 - – accumulated errors can become unmanageable
- Statistical spectrum analysis (J. Wang, ICCAD04, P. Ghanta, DATE'05)
 - solve for coefficients of orthogonal polynomials for statistical responses
 - Have not been applied to projection based model order reduction



New contributions

- The contributions of proposed statistical spectrum model order reduction technique:
 - Applying statistical spectrum method to calculate moments
 - Modified Krylov subspace based model order reduction technique
 - Monte Carlo sampling to generate order reduced variational model



Problem formulation

- In RC network, the modified nodal analysis (MNA) equation is

$$Gv(t) + C \frac{dv(t)}{dt} = Bu(t)$$

- Variational elements exist in G and C matrices
 - The MNA equation becomes

$$G(\xi)v(t) + C(\xi) \frac{dv(t)}{dt} = Bu(t)$$

- Modeling intra-die Gaussian uncorrelated variations.
- Variations caused by circuit parameters, such as metal wire, width, thickness, etc. and caused by transistor parameters, such as channel length, width, gate oxide thickness.

Statistical Spectral Analysis Method (Ghanem'90)



- A random variable $x(t, \xi)$ can be represented by a set of orthogonal polynomials $\gamma_i(\xi)$,

$$x(t, \xi) = \sum_{n=0}^{\infty} a_n(t) \gamma_n(\xi)$$

- Hermite Polynomials are the best expansion for normal random variables
 - An order p Hermit Polynomial is defined as

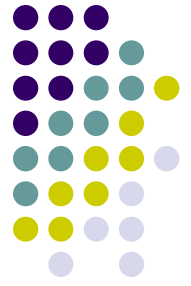
$$H_p(\{\alpha_1, \alpha_2, \dots, \alpha_p\}) = (-1)^p e^{\frac{1}{2}\xi' \xi} \frac{\partial^p}{\partial \alpha_1 \partial \alpha_2 \dots \partial \alpha_p} e^{-\frac{1}{2}\xi' \xi}$$

$\{\alpha_i\}$ are any set of p variables chosen from $\{\xi_1, \xi_2, \dots, \xi_n\}$ allowing repetition

- For $x(t, \xi)$, an order p Hermit Polynomial expansion is

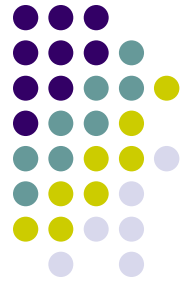
$$x(t, \xi) = \sum_{i=0}^N a_i(t) H_i(\xi)$$

Simulation Approach Based on Hermite PCs



- Solve
$$G(\xi)v(t, \xi) + C(\xi)\frac{dv(t, \xi)}{dt} = Bu(t)$$
- For independent random variables, $\{\xi_1, \xi_2\}$, $p=2$
 - $G(t, \xi)$, $C(t, \xi)$ can be written in Hermite polynomial chaos (PC)
$$G(t, \xi) = G_0(t) + G_1(t)\xi_1 + G_2(t)\xi_2 + G_3(t)(\xi_1^2 - 1) + G_4(t)(\xi_2^2 - 1) + G_5(t)\xi_1\xi_2$$
$$C(t, \xi) = C_0(t) + C_1(t)\xi_1 + C_2(t)\xi_2 + C_3(t)(\xi_1^2 - 1) + C_4(t)(\xi_2^2 - 1) + C_5(t)\xi_1\xi_2$$
 - ξ_1, ξ_2 are independent with normal distribution
 - How to solve for $v_i(t) \rightarrow \rightarrow$ through statistical moment computation

Hermite polynomial chaos



- Use a series of orthogonal polynomials to decompose Gaussian random processes.
- The basis of Hermite polynomial is expanded to second order in experiment. Higher order improves accuracy.
- Galerkin method (principle of orthogonality):
 - Error is minimized when the error is orthogonal to the approximation
- Calculate coefficients in deterministic form
- Mean and variance of random variables can be computed easily.

Projection Krylov Subspace MOR

- review



- A general linear circuit system

$$C\dot{x}_n = -Gx_n + Bu_m$$

$$i_m = Lx_n$$

- Transfer function in s domain

$$H(s) = L(G + sC)^{-1}B \quad \longrightarrow \quad H(s) = L(I - sA)^{-1}R$$
$$A = -G^{-1}C \quad R = G^{-1}B$$

- The block Krylov space

$$Kr(A, R, q) = \text{span}[R, AR, A^2R, A^3R, \dots]$$

$$V = \text{orthonomal}\{Kr(A, R, q)\}$$



Standard Krylov Subspace MOR

- Using Arnoldi or Lanczos process to orthonormalize the Krylov subspace

$$Kr(A, R, q) = span[R, AR, A^2 R, A^3 R, \dots]$$

- i th block moment is computed and orthonormalized **immediately** after generation against all the existing orthonormalized block moments.
- Congruence transformation to compute the reduced model

$$\hat{G} = V^T G V; \hat{C} = V^T C V; \hat{B} = V^T B$$

Modified Krylov subspace MOR



- Compute all the block moments first.

$$m_0 = G^{-1}B;$$

$$m_1 = -G^{-1}Cm_0;$$

...

- Orthonormalize moment vector to get projection matrix V by Gram-Schmidt method.

$$m_i = -G^{-1}Cm_{i-1}; \forall i > 0$$

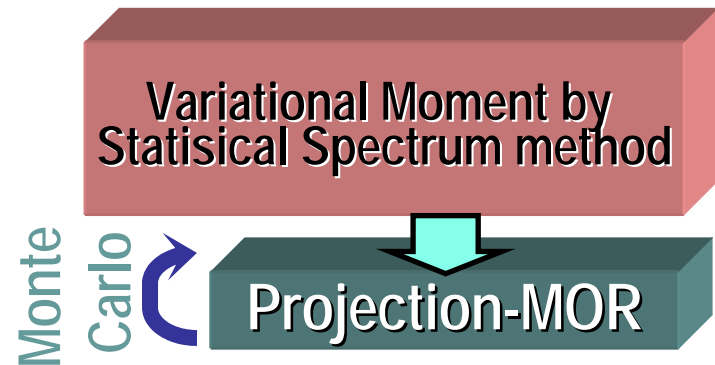
- Congruence transform $\hat{G} = V^T G V; \hat{C} = V^T C V; \hat{B} = V^T B$

Reason: To divide the statistical modeling process between moment computation and model generations



New statistical MOR method

- Compute all the moments using statistical spectrum method. (as moments can be computed by solving $Ax = b$)
- Compute all the reduced models using Monte Carlo methods from variational moments
 - Orthonormalization
 - Congruence transformation
 - Pole/residues computations



Galerkin method in Hermit PCs

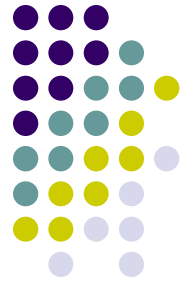


- In Galerkin method, the best approximation of v is obtained when the error is orthogonal to the approximation

$$\langle e(t, \xi), H_k(\xi) \rangle = 0$$

$$k = 0, 1, \dots, p$$

Where P is the order of Hermit PC



Example: one random variable

- Expand Hermite PC to first order ($p=1$), variational G and C becomes

$$G = g_0 + g_1 \xi; C = c_0 + c_1 \xi$$

- Expand Hermite PC to second order ($p = 2$) on moments

$$m_0 = [a_{m_0}, a_{m_1}, a_{m_2}] [1, \xi, \xi^2 - 1]^T;$$

$$m_{2q} = [a_0, a_1, a_2] [1, \xi, \xi^2 - 1]^T;$$

$$m_{2q-1} = [b_0, b_1, b_2] [1, \xi, \xi^2 - 1]^T;$$

- $[a_{m_0}, a_{m_1}, a_{m_2}]$, $[a_0, a_1, a_2]$, and $[b_0, b_1, b_2]$ are coefficients for m_0 , m_{2q} , and m_{2q-1} with respect to Hermite PC

Solution for one variable



- Applying Galerkin method, and the equalities in Gaussian distributions; the 0th moment is given by:

$$\begin{bmatrix} g_0 & g_1 & 0 \\ g_1 & g_0 & 2g_1 \\ 0 & 4g_1 & 4g_0 \end{bmatrix} \begin{bmatrix} a_{m0} \\ a_{m1} \\ a_{m2} \end{bmatrix} - \begin{bmatrix} B \\ 0 \\ 0 \end{bmatrix} = 0$$

- Likewise, the higher order moments are computed recursively

$$\begin{bmatrix} g_0 & g_1 & 0 \\ g_1 & g_0 & 2g_1 \\ 0 & 2g_1 & 2g_0 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix} - \begin{bmatrix} c_0 & c_1 & 0 \\ c_1 & c_0 & 2c_1 \\ 0 & 2c_1 & 2c_0 \end{bmatrix} \begin{bmatrix} b_0 \\ b_1 \\ b_2 \end{bmatrix} = 0$$

Solution for n random variables



- Similarly, the 0th moment can be expressed as:

$$\begin{bmatrix} g_0 & g_1 & g_2 & \dots & g_i & \dots & g_n \\ g_1 & g_0 & 0 & \dots & 0 & \dots & 0 \\ g_2 & 0 & g_0 & \dots & 0 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ g_i & 0 & 0 & \dots & g_0 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ g_n & 0 & 0 & \dots & \dots & \dots & g_0 \end{bmatrix} \begin{bmatrix} a_{m0} \\ a_{m1} \\ a_{m2} \\ \dots \\ a_{mi} \\ \dots \\ a_{mn} \end{bmatrix} - \begin{bmatrix} B \\ 0 \\ 0 \\ \dots \\ 0 \\ \dots \\ 0 \end{bmatrix} = 0$$

- The recursive moments can be computed as:

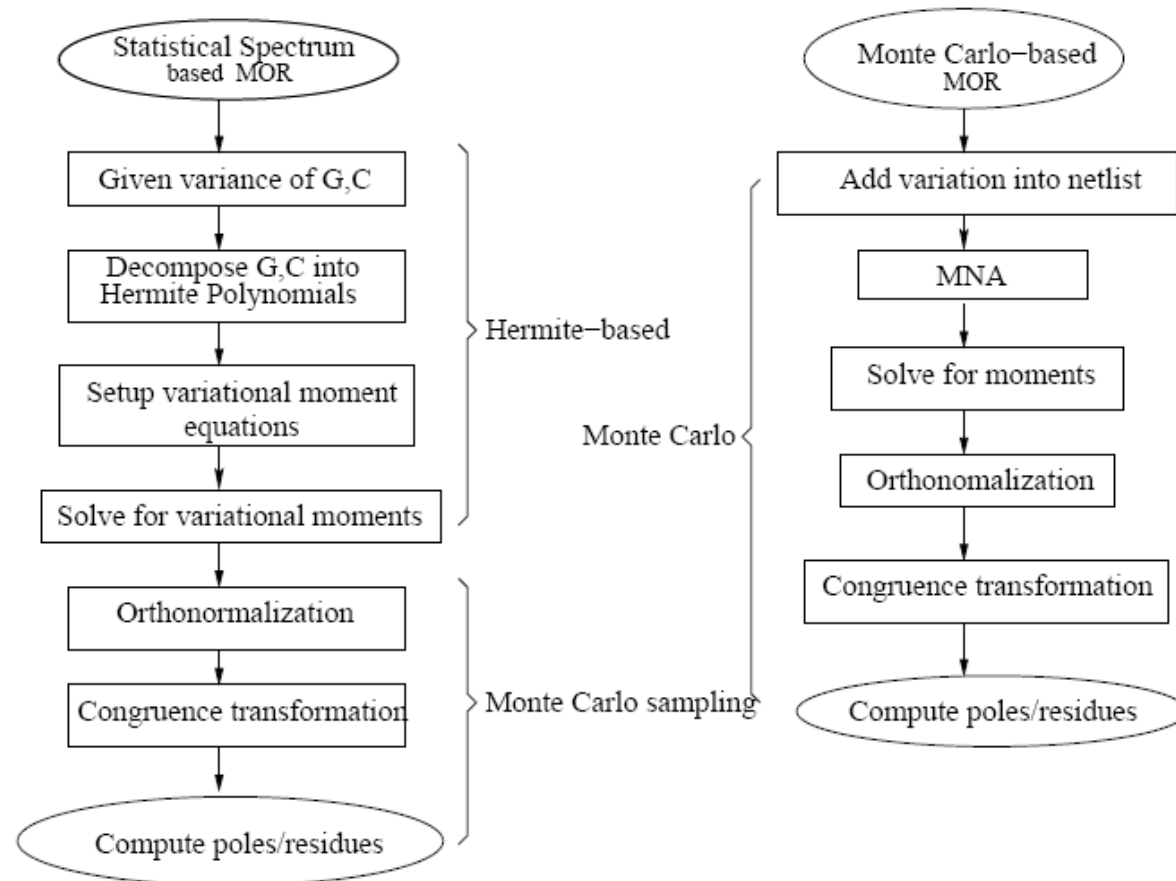
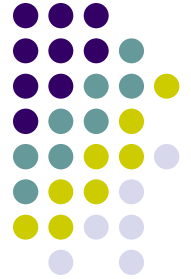
$$\begin{bmatrix} g_0 & g_1 & g_2 & \dots & g_i & \dots & g_n \\ g_1 & g_0 & 0 & \dots & 0 & \dots & 0 \\ g_2 & 0 & g_0 & \dots & 0 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ g_i & 0 & 0 & \dots & g_0 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ g_n & 0 & 0 & \dots & \dots & \dots & g_0 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ \dots \\ a_i \\ \dots \\ a_n \end{bmatrix} + \begin{bmatrix} c_0 & c_1 & c_2 & \dots & c_i & \dots & c_n \\ c_1 & c_0 & 0 & \dots & 0 & \dots & 0 \\ c_2 & 0 & c_0 & \dots & 0 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ c_i & 0 & 0 & \dots & c_0 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ c_n & 0 & 0 & \dots & \dots & \dots & c_0 \end{bmatrix} \begin{bmatrix} b_0 \\ b_1 \\ b_2 \\ \dots \\ b_i \\ \dots \\ b_n \end{bmatrix} = 0$$

Statistical Spectrum Model Order Reduction (SSMOR)



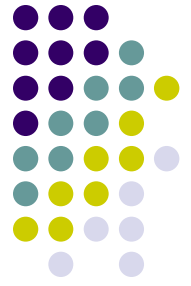
- Modified Krylov subspace model order reduction
- In each sampling, orthonormalize moment vectors thru Gram-Schmidt algorithm
- SSMOR computes variational block moments, then switch to Monte Carlo to generate reduced models
- Congruence transformation takes G and C into dimension-reduced matrices → **guarantee passivity**
- Use projection-based method to obtain poles and residues

SSMOR flowchart



Monte Carlo sampling on reduced models →→ much faster!

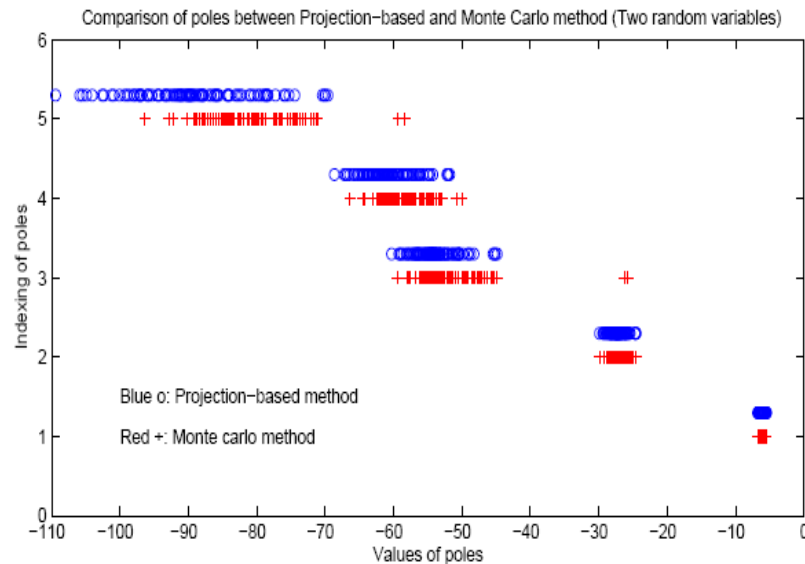
Experiment



- First q moments are obtained ($q=10$), to obtain 5 poles and residues.
- Compare accuracy between explicit moment matching, AWE, and modified Krylov subspace projection based method
- Smaller size for fast simulation, and test scalability in larger sized circuits
- The sample size is 2000 for Monte Carlo to guarantee roughly 99% confidence level
- Piecewise linear responses are obtained to compare the percentage of errors for deterministic and stochastic stimulus.

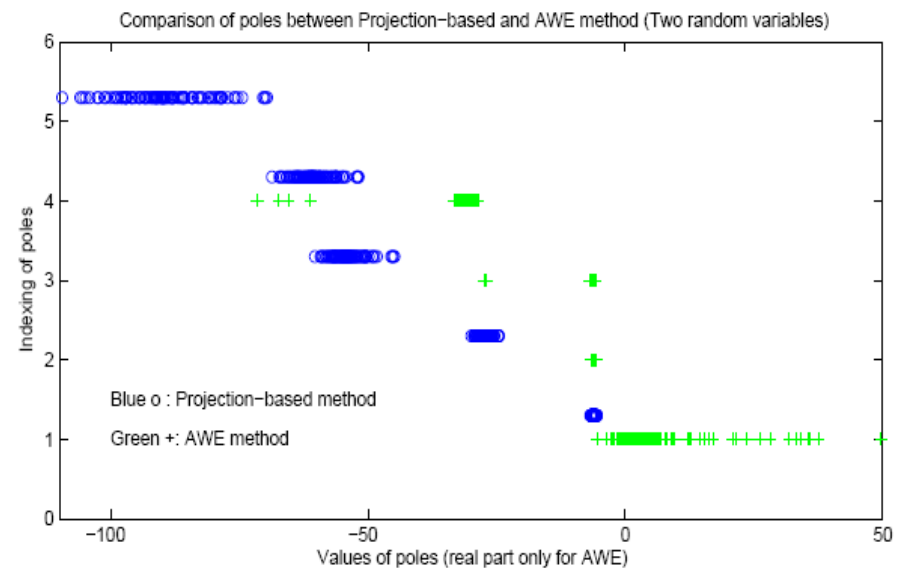


Projection based vs. AWE



Projection-based method matches well with Monte Carlo

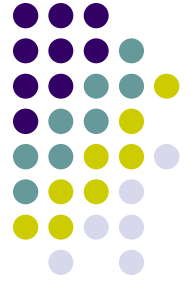
SSMOR is projection-based!



Pole placement does NOT converge!

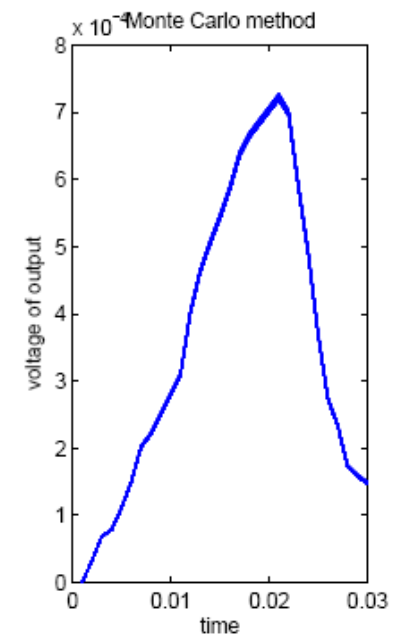
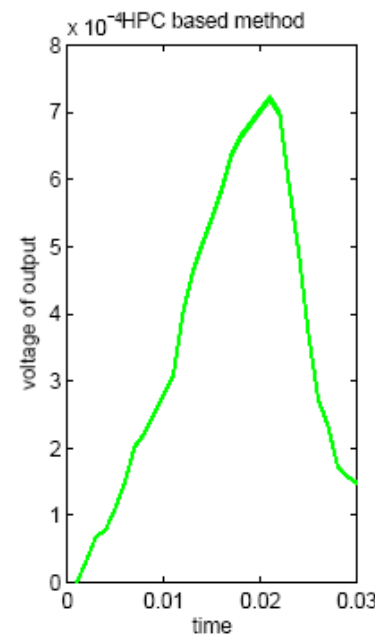
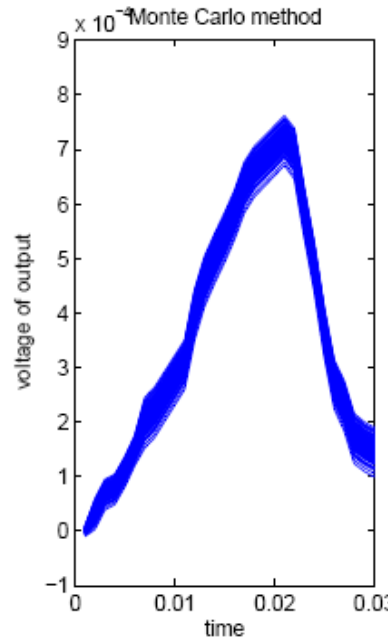
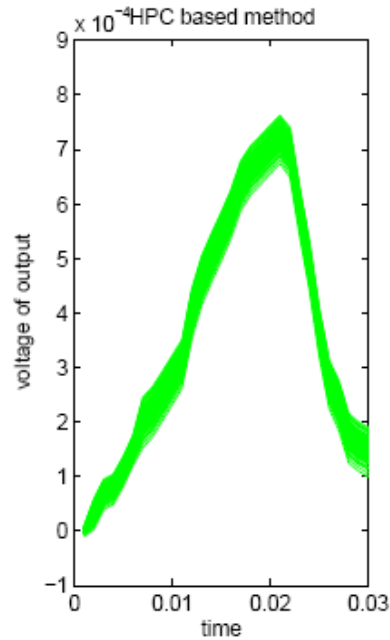
AWE is numerically unstable

Piecewise linear responses (SSMOR)



Stochastic stimulus

Deterministic stimulus



Responses are almost identical in comparison to Monte Carlo!



SSMOR vs. Monte Carlo

Comparison of voltage response → → less than 1% of errors

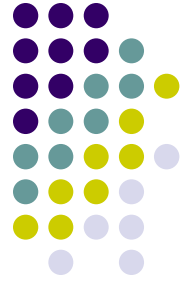
Time instance (e-3) s	SSMOR		MC		% error	
	mean (e-5)	std (e-6)	mean (e-5)	std (e-6)	mean %	std %
3	6.852	9.518	6.835	9.487	0.25	0.327
5	11.11	9.489	11.078	9.454	0.253	0.369
20	70.27	15.56	70.1	15.45	0.232	0.732

	#node	SSMOR	MC	Speedup
Ckt1	33	1	37.03	37 times
Ckt2	553	1	162.16	162 times
Ckt3	1720	1	118.03	118 times

**SSMOR method
is much faster
without losing
accuracy!**

Runtime comparison between → → 100X speedup

Conclusion



- Statistical Spectrum Model Order Reduction (SSMOR) is suitable to consider inter-die and intra-die variations
- Fast analysis on interconnect, such as power grid or clock tree networks, with stochastic stimulus without losing accuracy
- Order of faster than Monte Carlo methods with reasonable accuracy.
- AWE is numerically unstable; projection-based method is used in SSMOR and was shown to be very robust.
- Need to consider spatial correlations in the future.