An Extended SVD-based Terminal and Model Order Reduction Algorithm

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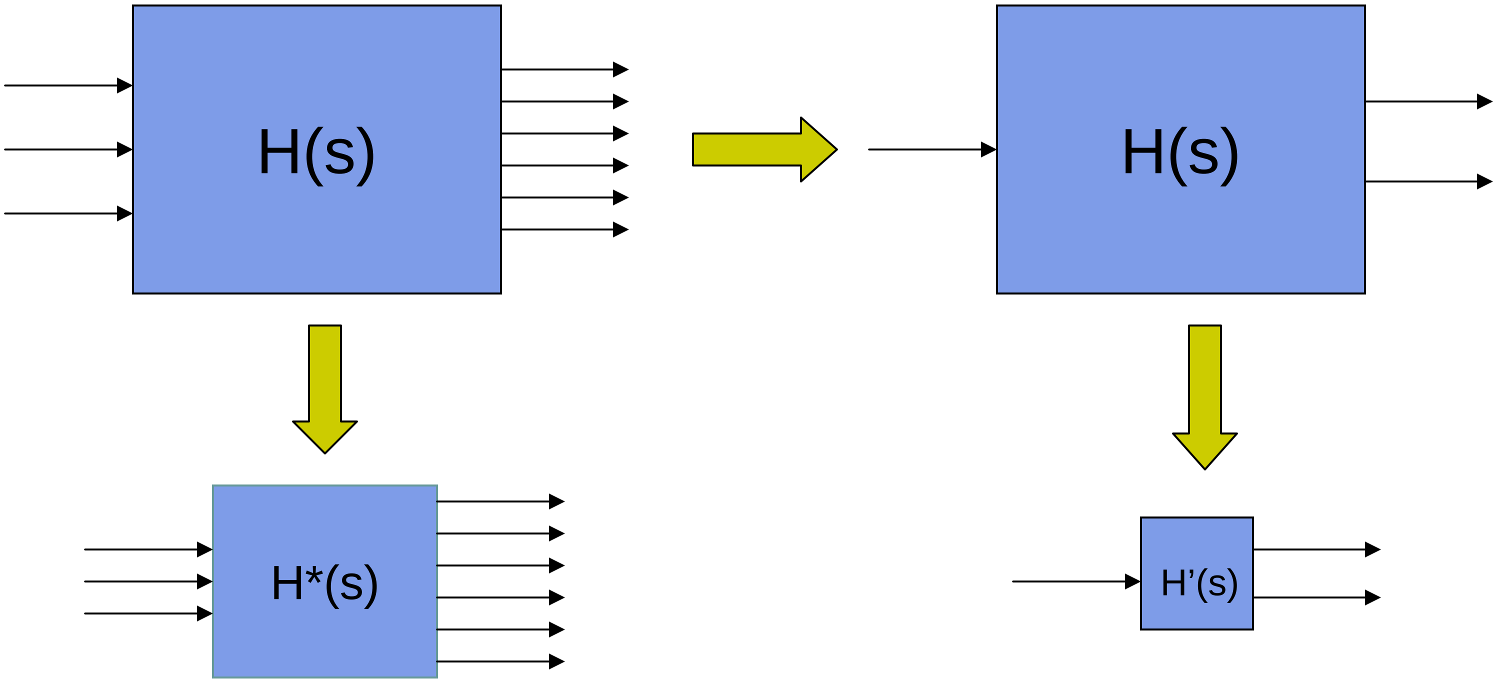
Outline

- Motivation of terminal and model order reduction
- Review of basic SVDMOR method
- The new extended-SVDMOR (ESVDMOR) algorithm
  - Input and output moment matrices
  - The SVD-based new terminal reduction method
  - ESVDMOR reduction flow
- Experimental results
- Summary

Why Terminal Reduction?



Terminal reduction leads to more compact models!

Projection Krylov Subspace

- A general linear circuit system

$$C\dot{x}_n = -Gx_n + Bu_m$$

$$i_m = Lx_n$$

- Transfer function in s domain

$$H(s) = L(G + sC)^{-1}B$$

$$A = -G^{-1}C \quad R = G^{-1}B$$

$$\Rightarrow \quad H(s) = L(I - sA)^{-1}R$$

- The block Krylov space $R \subseteq R^{n \times p}$

$$Kr(A, R, q) = span[R, AR, A^2R, A^3R, \ldots]$$

$$P = orthonomal\{Kr(A, R, q)\}$$

Review of SVDMOR Algorithm

A time domain circuit equations

$$C\dot{x}(t) + Gx(t) = Bu(t)$$
$$y(t) = Lx(t)$$

Terminal reduction via SVD:

$$m_0 = U\Sigma V^T \approx U_k \Sigma_k V_k^T$$

$$B = B_b V_k^T \qquad B_b = BV_k (V_k^T V_k)^{-1}$$
$$L^T = L_c U_k^T \qquad L_c = L^T U_k (U_k^T U_k)^{-1}$$

Its transfer functions in s domain:

$$H(s) = L(G + sC)^{-1} B$$

$$m_0 = LG^{-1}B$$
$$m_1 = LG^{-1}CG^{-1}B$$
$$\vdots$$
$$m_i = L(G^{-1}C)^{-1}G^{-1}B$$

Only consider DC response (DC moment) !

SVDMOR method (cont.)

$$H(s) \approx U_k L_c^T (G + sC)^{-1} B_b V_k^T$$

$$H_r(s) = L_c^T (G + sC)^{-1} B_b$$

$$\hat{H}_r(s) = \hat{L}_c^T (\hat{G} + s\hat{C})^{-1} \hat{B}_b$$

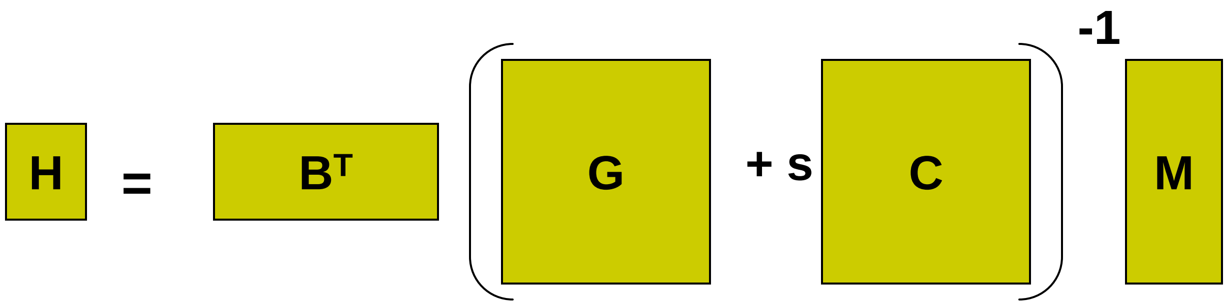
$$H(s) = U_k \hat{L}_c^T (\hat{G} + s\hat{C})^{-1} \hat{B}_b V_k^T$$

where

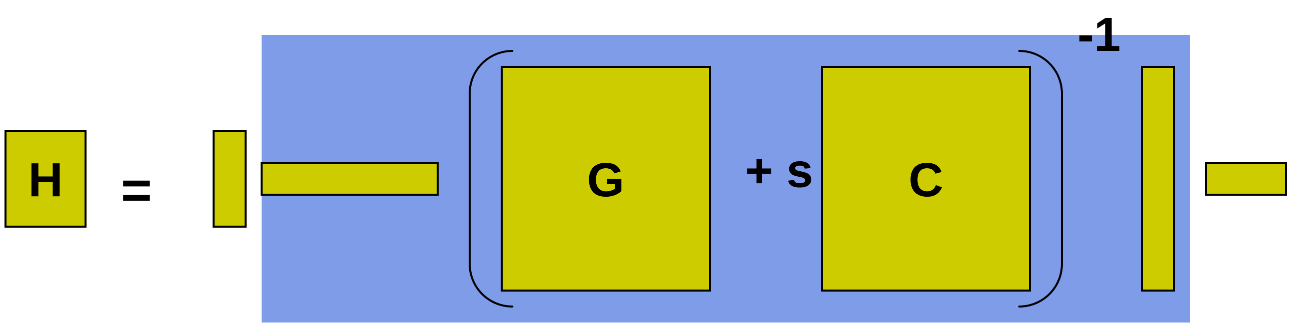
$$\hat{G} = V^T G V; C = V^T C V;$$

$$\hat{B}_b = V^T B_b; \hat{L}_c^T = L_c^T V$$

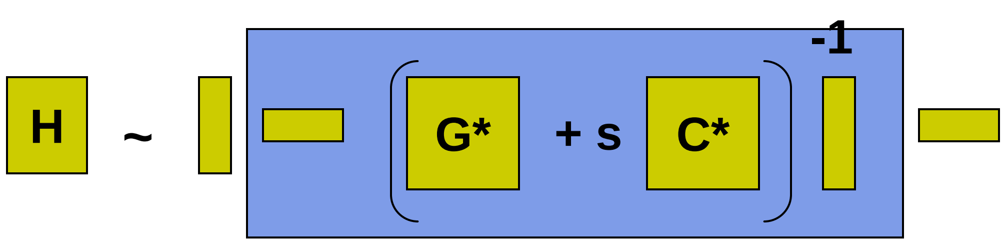
SVDMOR Algorithm (cont.)



After SVD on boundary matrices



After projection MOR



Limitations of SVDMOR

- SVD is performed only on DC response or particular order of moments.
- Input and output terminals are reduced at the same time.
  - Mapping can lead to large errors due to different ranks of input and output matrices.
- The SVDMOR does not always lead to passive models.

Block Moment Details

- The (i)th moment block

$$m_i = \begin{bmatrix} m^i_{1,1} & m^i_{1,2} & \cdots & m^i_{1,p} \\ m^i_{2,1} & m^i_{2,2} & \cdots & m^i_{2,p} \\ \vdots & \vdots & \vdots & \vdots \\ m^i_{q,1} & m^i_{q,2} & \cdots & m^i_{q,p} \end{bmatrix}$$

Moment Matrix Formulation

- Output moment matrix

$$M_O = \begin{bmatrix} m_0^T \\ m_1^T \\ \vdots \\ m_{r-1}^T \end{bmatrix}$$

- Each column represents a moment series of output due to all input's stimuli

- Input moment matrix

$$M_I = \begin{bmatrix} m_0 & m_1 & \cdots & m_{r-1} \end{bmatrix}$$

- Each column represents a moment series at all outputs due to an input stimulus

New Terminal Reduction

- Find projection subspace for terminal reduction

$$M_I = U_I \Sigma_I V_I^T \approx U_{I_{ki}} \Sigma_{I_{ki}} V_{I_{ki}}^T$$

$$M_O = U_O \Sigma_O V_O^T \approx U_{O_{ko}} \Sigma_{O_{ko}} V_{O_{ko}}^T$$

$$B = B_r V_{I_{ki}}^T$$

$$L = V_{O_{ko}} L_r$$

$$B_b = B V_{I_{ki}} (V_{I_{ki}}^T V_{I_{ki}})^{-1}$$

$$L_c = L^T U_{O_{ko}} (U_{O_{ko}}^T U_{O_{ko}})^{-1}$$

Notice that row in $V_{I_{ki}}, U_{O_{ko}}$ are orthogonal

$$V_{I_{ki}}^T V_{I_{ki}} = I, U_{O_{ko}}^T U_{O_{ko}} = I$$

$$B_b = B V_{I_{ki}}$$

$$L_c = L^T U_{O_{ko}}$$

New Terminal Reduction (cont'd)

- Terminal reduced subsystem

$$H(s) = V_{O_{ko}} L_r (G + sC)^{-1} B_r V_{I_{ki}}^T$$

$$H_r(s) = L_r (G + sC)^{-1} B_r$$

- Model order reduction and terminal recovery

$$\hat{H}_r(s) = \hat{L}_r (\hat{G} + s\hat{C})^{-1} \hat{B}_r$$

$$\hat{H}(s) = V_{O_{ko}} \hat{H}_r(s) V_{I_{ki}}^T$$

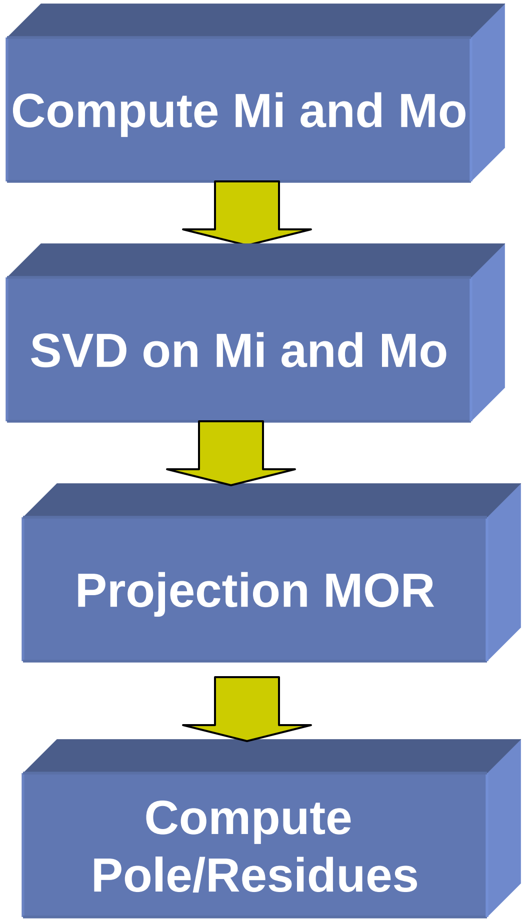
$$where$$

$$\hat{G} = V^T G V; C = V^T C V$$

$$\hat{B}_r = V^T B V_{I_{ki}}; \hat{L}_r = V_{O_{ko}}^T L B V$$

ESVDMOR Algorithm Flow

i. Compute multiple block moments
ii. Construct input and output moment matrices respectively
iii. Perform SVD-based low rank approximation to input and output position matrices
iv. Apply normal projection Krylov subspace based model order reduction on terminal reduced subsystem
v. Computer the final transfer function of the reduced system



Passivity Discussion

- SVDMOR or ESVDMOR does NOT lead to passive models

$$C\dot{x}_n = -Gx_n + Bu_m$$
$$i_m = Lx_n$$

  - Passive projection require $B^T = L$
  - In SVDMOR-like methods, it becomes $B_r^T = L_r$
    - The moment matrix M or M_i/M_o must be symmetric

      The interconnects are driven by the same source types
  - When $B^T = L$, i.e. all the terminals are treated as both input and output terminals, the moment matrix typically are close to full rank.
    - We can't perform the terminal reduction at all.

Example

- Treat all terminals of net1026 as bi-direction

#	m_0	m_1
1	0.58970	7.5175
2	0.55093	0.33515
3	0.50215	0.30440
4	0.41875	0.28897
-	---	---
-	---	---
260	0.00059548	0.0099117
261	0.0001499	0.0098567
262	2.9535e-17	0.0096795

Example 1

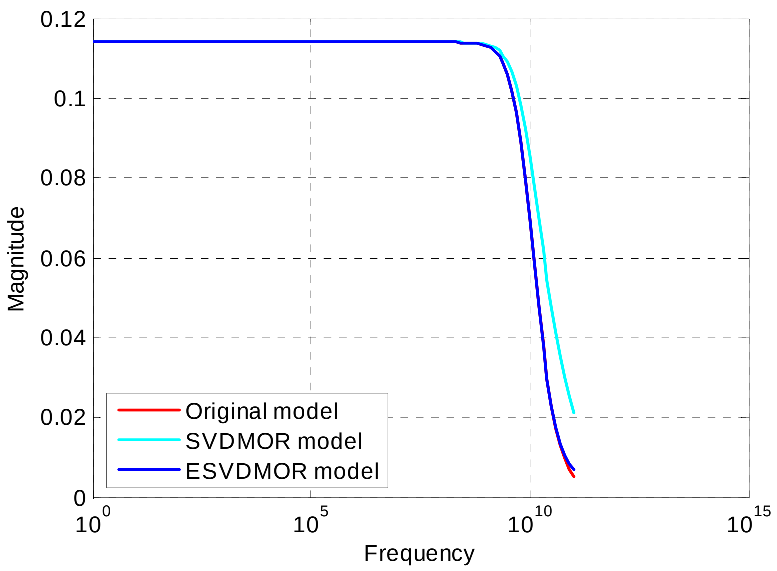
- Net27 (14 inputs, 118 outputs)

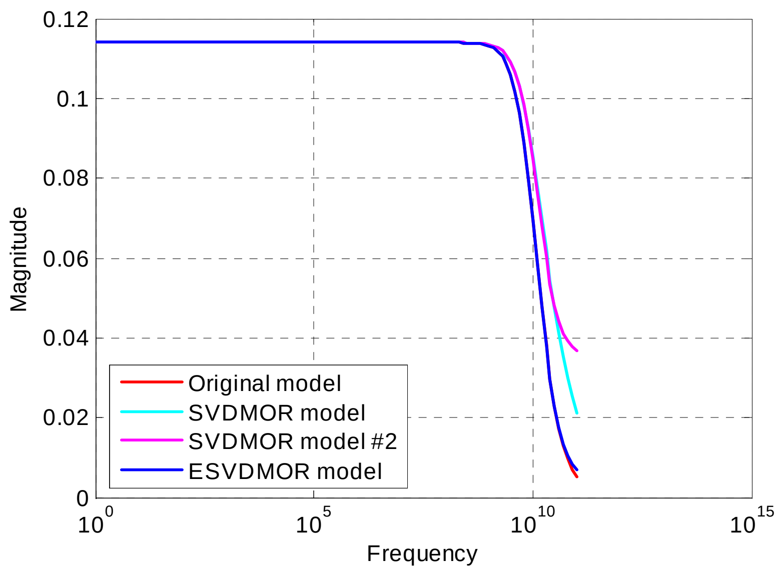
#	m_0	M_I	M_O
1	5.1587	5.1587	19.828
2	3.9883e-14	3.9883e-14	4.4677
3	1.6681e-14	1.6681e-14	1.6517
4	---	---	0.3045
5	---	---	0.0348
6	---	---	2.4611e-3
7	---	---	1.6134e-4

Table 1: The singular values of DC moment, Input moment matrix and Output moment matrix of the circuit net27.

Example 1 (cont.)

- Net27 (14 inputs, 118 outputs)





SVDMOR with 1 input 1 output; ESVDMOR with 1 input 5 output; SVDMOR#2 with 5 input 5 output.

Example 2

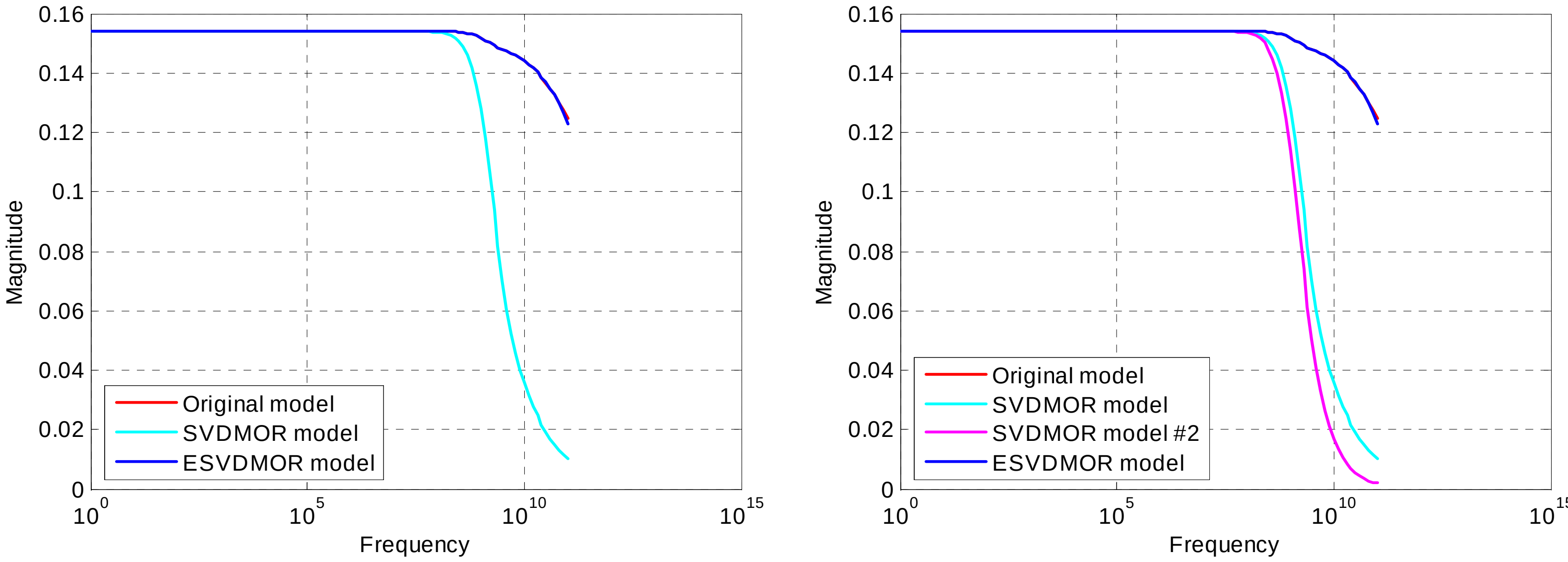
- Net1026 (6 inputs, 256 outputs)

#	m_0	M_I	M_O
1	7.5175	7.5175	2097.6
2	2.7376e-15	2.7376e-15	10.501
3	7.5742e-16	7.5742e-16	1.6625
4	---	---	0.12577
5	---	---	0.00134
6	---	---	8.278e-6
7	---	---	3.9216e-8

Table 2: The singular values of DC moment, Input moment matrix and Output moment matrix of the circuit net1026.

Example 2 (cont.)

- Net1026 (6 inputs, 256 outputs)



SVDMOR with 1 input 1 output; ESVDMOR with 1 input 5 output; SVDMOR#2 with 5 input 5 output.

Example 2 (cont.)

- Treat all terminals of net1026 as bi-direction

#	m_0	m_1
1	0.58970	7.5175
2	0.55093	0.33515
3	0.50215	0.30440
4	0.41875	0.28897
-	---	---
-	---	---
260	0.00059548	0.0099117
261	0.0001499	0.0098567
262	2.9535e-17	0.0096795

Example 3

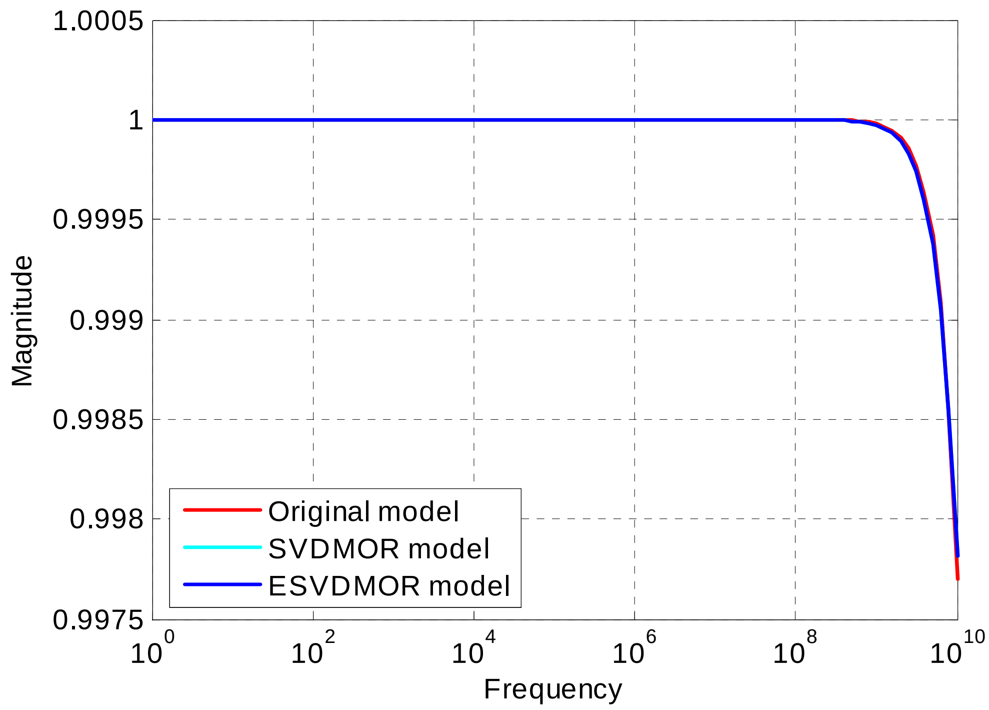
- Net55 (59 inputs, 31 outputs)

#	m_0	M_I	M_O
1	4.224	4.224	4.224
2	2.4495	2.4495	2.4495
3	0.4126	0.4126	0.4126
4	5.5185e-16	3.1765e-13	5.5185e-16

Table 4: The singular value of DC moment, Input moment matrix and Output moment matrix of the circuit net55.

Example 3 (cont.)

- Net55 (59 inputs, 31 outputs)



SVDMOR, ESVDMOR, and SVDMOR#2 are all with 3 input 3 output.

Summary

- SVDMOR only consider DC moment.
- Moment matrices include more information.
- ESVDMOR use high order moments to exploit the appropriate number of independent input and output terminals separately.
- SVD based terminal reduction methods are ineffective when passivity is considered.

Thanks & Questions?